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II

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Chinese-English

四元玉鉴

Jade Mirror of the Four Unknowns

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陈在新 英译
郭金海 整理

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1.

【原文】

今有直邑不知大小，各中开门。只云：南门外二百四十步有塔，人出西门行一百八十步见塔，复抹邑西南隅行一里二百四十步，恰至塔所。问：邑长、阔各几何？

答曰：长一里一百二十步，阔一里。

术曰：立天元一为邑半长，如积求之，得一十八亿六千六百二十四万为正实，一千五百五十五万二千为从方，二十七万为益上廉，四百八十为从下廉，一为正隅，三乘方开之，[1]得二百四十步。倍之，即长。

又：立天元一为邑半阔，如积求之，得一十八亿六千六百二十四万为正实，二千七十三万六千为从方，二十七万为益上廉，三百六十为从下廉，一为正隅，三乘方开之，[2]得一百八十步。倍之，即阔。合问。

【注释】

[1] 开方式的现代形式为：$x^4 + 480x^3 - 270000x^2 + 15552000x + 1866240000 = 0$。(陈)

[2] 开方式的现代形式为：$x^4 + 360x^3 - 270000x^2 + 20736000x + 1866240000 = 0$。(陈)

【今译】

今有长方形的城，不知大小，各在城墙的中间开门。只云：南门外240步有一座塔，人出西门西行180步，刚刚见到塔；此人擦过城的西南角走1里240步，恰好到塔的所在地。问：城的长、阔各为多少？

答：长1里20步，阔1里。

术：设天元一为城的长的$\frac{1}{2}$，以如积方法求其解。得到1866240000作为常数项，15552000作为一次项系数，-270000作为二次项系数，

Gou Gu Ce Wang (Surveying with Right Triangles)

8 Problems

1. A city built in the form of a rectangle has a gate at mid-point of each side of the wall. At a distance of 240 *bu* outside of the south gate stands a pagoda. A person went out from the west gate and after walking 180 *bu* saw the pagoda. He turned and walking straight toward it found the distance to be 1 *li* and 240 *bu*. Find the length and the width of the city.

Ans. Length of the wall, 1 *li* 120 *bu*;

width of the wall, 1 *li*.

Process. Let the element *tian* be one-half of the length of the city wall. From the statement we have 1866240000 for the positive *shi*, 15552000 for the positive *fang*, 270000 for the negative upper *lian*, 480 for the positive lower *lian*, and 1 for the positive *yu*, an expression [1] of the fourth degree whose root, 240 *bu*, is the required number. Double this root is the length of the city wall. Again let the element *tian* be one-half of the width of the city wall. From the statement we have 1866240000 for the positive *shi*, 20736000 for the positive *fang*, 270000 for the negative upper *lian*, 360 for the positive lower *lian*, and 1 for the positive *yu*, a biquadratic expression[2] whose root, 180 *bu*, is the required number. Double this root is the width of the city wall.

480作为三次项系数，1作为最高次项系数，开四次方，得到240步。加倍，就是长。　又：设天元一为城的阔的$\frac{1}{2}$，以如积方法求其解。得到1866240000作为常数项，20736000作为一次项系数，−270000作为二次项系数，360作为三次项系数，1作为最高次项系数，开四次方，得到180步。加倍，便得到阔。符合所问。

2.

【原文】

今有圆城不知大小，各中开门。甲、乙俱从城心而出。甲出南门一十五步而立，乙出东门四十步见甲。[1] 问：城周几何？

答曰：一里。

术曰：立天元一为城之半圆径，如积求之，得三十六万为正实，六万六千为从方，二千四百为从上廉，一为益隅，三乘方开之，[2] 得半圆径六十步。倍而三之[3]，即城周。合问。

【注释】

[1] 圆城心与甲、乙立处形成一个勾股形，则 $a = r + 15$，$b = r + 40$。（郭）

[2] 开方式的现代形式为：$-x^4 + 2400x^2 + 66000x + 360000 = 0$。（陈）

[3] 取 $\pi = 3$，故“三之”。（郭）

【今译】

今有圆城，不知大小，在城墙的东西南北中间开门。甲、乙二人同时从圆城的中心出发，甲出南门15步站住。乙出东门40步，恰好见到甲。问：圆城的周长为多少？

答：1里。

术：设天元一为城的半径，以如积方法求其解。得到360000作为常数项，66000作为一次项系数，2400作为二次项系数，−1作为最高次项系数，开四次方，得到圆半径60步。加倍，乘以3，就是圆城周长。符合所问。

[Notes]

[1] The expression in modern form is the equation: $x^4 + 480x^3 - 270000x^2 + 15552000x + 1866240000 = 0$. (C)

[2] The expression in modern form is the equation: $x^4 + 360x^3 - 270000x^2 + 20736000x + 1866240000 = 0$. (C)

2. A city, built in the form of a circle, has four gates opening to the west, east, north, and south. Jia and Yi both started from the center of the city. *Jia* went out from the south gate and stopped at a distance of 15 *bu*. *Yi* went out from the east gate and after walking 40 *bu*, saw *Jia* at a distance. [1] Find the circumference of the city.

Ans. 1 *li*.

Process. Let the element *tian* be the radius of the city. From the statement we have 360000 for the positive *shi*, 66000 for the positive *fang*, 2400 for the positive upper *lian*, and 1 for the negative *yu*, an expression [2] of the fourth degree whose root, 60 *bu*, is the required radius. The radius multiplied by 3 gives the circumference of the city [3].

[Notes]

[1] A right triangle is formed by the center of the city, the places that Jia and Yi stopped. Then, $a=r+15$, $b=r+40$. (G)

[2] The expression in modern form is the equation: $-x^4 + 2400x^2 + 66000x + 360000=0$. (C)

[3] Use 3 for π, so the radius multiplied by 3. (G)

3.

【原文】

今有方城不知大小，各中开门。北门外九十步有邮亭一所，人于城中出西门外行一百六十步，却遥望参城隅见亭。[1] 问：城方几何？

答曰：二百四十步。

术曰：立天元一为城之半方面，如积求之，得一万四千四百为益实，一为正隅，平方开之，[2] 得一百二十步。倍之。合问。

【注释】

[1] 此即邮亭、城之西北隅与人出西门遥望处三点共线。它与城之北门、西门形成两个勾股形，其勾分别为90、方城边长之半，其股分别为方城边长之半、160。(郭)

[2] 开方式的现代形式为：$x^2 - 14400 = 0$。(陈)

【今译】

今有正方形的城，不知大小，在四面城墙的正中开门。北门外90步处有一所邮亭，有人从城中心出西门，走160步，恰好与城之西北角、邮亭三点共线。问：城的边长为多少？

答：240步。

术：设天元一为城的边长的 $\frac{1}{2}$，以如积方法求其解。得到 -14400 作为常数项，1作为最高次项系数，开平方，得到120步。加倍。符合所问。

4.

【原文】

今有圆城不知高、远。立两表各高一丈二尺，表间相去八十尺，令前表与后表参相直。于前表退行六十尺，人目薄地遥望乳头，与前表末参合。又从后表退行一百尺，人目薄地遥望乳头，与后表末参合。[1] 问：城高

3. A city built in the form of a square has a gate at the mid-point of each side of the wall. An arbor stands 90 *bu* from the north gate. A person started from the center of the city and after walking 160 *bu* beyond the west gate saw the arbor in the straight line with the corner of the city.[1] Find a side of the city.

Ans. 240 *bu*.

Process. Let the element *tian* be one-half of a side of the city. From the statement we have 14400 for the negative *shi*, and 1 for the positive *yu*, a quadratic expression[2] whose root, 120 *bu*, is the required number. Double this root is a side of the city.

【Notes】

[1] The arbor, the northwest corner, and the place where the person saw the arbor after he went out the west gate are in a straight line. Two right triangles are formed by the line, the north and west gates. Their *gou* are 90 and the half side of the city. Their *gu* are the half side of the city and 160. (G)

[2] The expression in modern form is the equation: $x^2 - 14400 = 0$. (C)

4. The distance of a round city and the height of its wall are unknown. Two measuring standards, 12 *chi* in length, are erected in a straight line with the city, the distance between them being 80 *chi*. At a point 60 *chi* from the first standard the surveyor lies on the ground and sees the tops of the wall and the first standard in a straight line. At a point 100 *chi* from the second standard he

及前表去城各几何?

答曰：城高三丈六尺，表去城一百二十尺。

术曰：立天元一为城高，如积求之，得一千四百四十为正实，四十为益方。上实，下法而一，[2] 得城高。 求表去城者，以前表退行乘表间为实，两表退行差为法，实如法而一。[3] 合问。

【注释】

[1] 此为用重差术的重表法求圆城之高、远。记测望物的高、远、两表间距、表高、前表退行（或日影）、后表退行（或日影）分别为 h, l, d, a, b_1, b_2，刘徽在《九章算术注》序中提出公式 $h=\frac{da}{b_2-b_1}+a$。（郭）

[2] 开方式的现代形式为：$-40x+1440=0$。（陈）

[3] 刘徽提出求远公式：$l=\frac{b_1d}{b_2-b_1}$。此即应用求远公式。（郭）

【今译】

今有圆城，不知其高、远。树立两支表，高各为1丈2尺，表间相距80尺，令前表、后表与城三者共线。从前表退行60尺，人目贴地遥望城头，恰与前表端三者共线。又从后表退行100尺，人目贴地遥望城头，恰与后表端三者共线。问：城高及前表距城各为多少?

答：城高3丈6尺，前表距城120尺。

术：设天元一为城高，以如积方法求其解。得到1440作为常数项，−40作为一次项系数。常数项作为被除数，一次项系数作为除数，除之，得到城高。 如果求前表距城，以前表退行乘表间相距作为被除数，两表退行之差作为除数，除之，就是前表距城。符合所问。

sees the tops of the wall and the second standard in a straight line. [1] What is the height of the wall and the distance of the first standard from the city?

Ans. Height of the wall, 36 *chi*;

distance from the first standard, 120 *chi*.

Process. Let the element *tian* be the height of the city wall. From the statement we have 1440 for the positive *shi* and 40 for the negative *fang*, a linear expression[2] whose root is the required height. To find the distance of the first standard from the city, use the product of the distance between the standards and the distance of the point of the observer from the first standard as the dividend, and the difference between the distance of the second point of the observer from the second standard, and the first point of the observer from the first standard as the divisor.[3] The quotient is the required number.

[Notes]

[1] It means that the height of the wall and the distance from the first standard were solved by the *chong biao fa* (the metod of two sandards or gnomons) of the *chong cha* method(the method of double differences). Let the height of the wall be h, the distance from the first standard l, the distance between the two standards d, the height of the standard a, the first standard backward (or sun shadow) b_1, and the second standard backward (or sun shadow) b_2. The formula given by Liu Hui in the introduction of *The Nine Chapters of Mathematical Procedures* is as follows: $h = \frac{da}{b_2 - b_1} + a$. (G)

[2] The expression in modern form is the equation: $-40x + 1440 = 0$. (C)

[3] Liu Hui gave the formula for solving the distance: $l = \frac{b_1 d}{b_2 - b_1}$. The formula is used here. (G)

5.

【原文】

今有方城，上有戍楼，不知高、远。立两表，齐高一丈五尺，表间相去八十步，令前表与后表参相直。人目高四尺，于前表退行三十步，遥望楼岑，与前表末参合。复望楼足，入表五尺六寸。又从后表退行五十步，遥望楼岑，与后表末参合。[1] 问：城、楼各高几何？

答曰：楼高二丈八尺，城高三丈一尺。

术曰：立天元一为楼高，如积求之，得二千八百为益实，一百为从方。开无隅平方而一，[2] 得楼高。 求城高者，置表高，减人目及入表，余，乘表间为实。以两表退行相多为法。实如法而一。合问。[3]

【注释】

[1] 此问是刘徽《海岛算经》之望松类问题，楼高即松高。记松高、山高、表间、表高、前表退行、后表退行、人目高、入表分别为 h, H, d, a, b_1, b_2, c_1, c_2，刘徽提出松高公式：$h = \dfrac{dc_2}{b_2 - b_1} + c_2$。（郭）

[2] 开方式的现代形式为：$100x - 2800 = 0$。（陈）

[3] 此即：$H = \dfrac{d(a - c_1 - c_2)}{b_2 - b_1}$。（郭）

【今译】

今有正方形的城，上面有戍楼，不知其高、远。树立两支表，高同为1丈5尺，表间相距80步，令前表、后表与城三者共线。人目高4尺。从前表退行30步，遥望楼顶，恰与前表端三者共线。再望楼底，入表5尺6寸。又从后表退行50步，遥望楼顶，恰与后表端三者共线。问：城高、楼高各为多少？

答：楼高2丈8尺，城高3丈1尺。

术：设天元一为楼高，以如积方法求其解。得到 −2800 作为常数项，

5. The height of a city wall built in the form of a square, the watch tower on the top of the wall, and the distance of the city are unknown. Two measuring standards, 15 *chi* in length, are erected in a straight line with the city wall at a distance of 80 *bu*. After walking back 30 *bu* from the first standard the surveyor sees the tops of the tower and the standard in a straight line, the distance of his eyes from the ground being 4 *chi*. From the same point of observation he looks at the foot of the tower and the line of sight cuts 5 *chi* 6 *cun* from the standard. Walking back 50 *bu* from the second standard he finds the tops of the tower and the standard are in a straight line with his eyes. [1] Find the height of the tower and the city wall.

Ans. Height of the tower, 28 *chi*;

height of the wall, 31 *chi*.

Process. Let the element *tian* be the height of the watch tower. From the statement we have 2800 for the negative *shi* and 100 for the positive *fang*, a linear expression[2] whose root is the height of the tower. To find the height of the city wall, subtract from the height of the standard the sum of the distance of the eyes from the ground and the length cut from the standard, then multiply the remainder by the distance between the two standards. Dividing this result by difference between the distance of the first point from the first standard and the second point from the second standard. Add to the quotient the difference between the height of the standard and the part cut off. The result is the desired height. [3]

100作为一次项系数，开无隅平方，得到楼高。 如果求城高，布置表高，减去人目高与入表，其余数乘表间相距，作为被除数，两表退行之差作为除数，除之，即得。符合所问。

6.

【原文】

今有方城，不知大小。立两表，东西相去四十三步二分。齐人目，以索连之，令东表与城东南隅及东北隅参相直。于东表退行一十四步八分，遥望城西北隅，入索东端一十步。又却北行，去表六十四步八分，遥望城西北隅，适与西表末相参合。[1] 问：城方、表去城[2] 各几何？

答曰：城方六里三百四十步，去表一十里八十五步五分步之一。

术曰：立天元一为城方，如积求之，得五千为正实，二为益方。上实，下法而一，得城方。[3] 求表去城者，入索乘北行去表，以两表相去除之，得为景差。内减东去表，余，以为法。又北行去表内减景差，余，乘东表退行，为实。实如法而一，即表去城之远。[4] 合问。

【注释】

[1] 此问是刘徽《海岛算经》之望邑类问题。记城方、表去城、景差、表间、东表退行、西表退行、入索分别为a，l，D，d，b_1，b_2，c，刘徽提出景差和城方公

【Notes】

[1] This is the sort of problem of *wang song* (surveying the pine) of Liu Hui's *Hai Dao Suan Jing* (Sea Island Mathematical Manual). The height of a tower is the height of a pine. Let the height of the pine be h, the height of a mountain H, the distance between the two standards d, the height of the standard a, the first standard backward b_1, the second standard backward b_2, the distance of a person's eyes from the ground c_1, and the line of sight cuts from the standard c_2. The formula for solving the height of the tower is as follows: $h = \frac{dc_2}{b_2 - b_1} + c_2$. (G)

[2] The expression in modern form is the equation: $100x - 2800 = 0$. (C)

[3] That is, $H = \frac{d(a - c_1 - c_2)}{b_2 - b_1}$. (G)

6. The size of a city which is in the form of a square is unknown. Two standards, in an east and west line, set 43 *bu* 2 *fen* apart, are connected by a rope tied at the height of the observer. The east standard is in a straight line with the southeast and the northeast corners of the wall. Walking backward from the east standard 14 *bu* 8 *fen* and looking at the northwest corner the line of vision cuts the rope 10 *bu* from its east end; walking 64 *bu* 8 *fen* farther north the west standard falls within the line of vision. [1] Find a side of the city and its distance from the standards [2].

Ans. A side of the city, 6 *li* 340 *bu*;

distance from the standards, 10 *li* $85\frac{1}{5}$ *bu*.

Process. Let the element *tian* be a side of the city. From the statement we have 5000 for the positive *shi* and 2 for the negative *fang*, a linear expression [3] whose root is the required side. To find the distance of the city from the standard multiply the length of the rope cut by the first line of vision by the distance of the observer's second position from the standard and divide by the distance between the two standards. This quotient is the

式：$D=\frac{cb_2}{d}$。$a=\frac{c(b_2-b_1)}{c-a}$。（郭）

［2］“表去城”，原文作“去表去城”，或“表”前衍“去”字，或“去表”后衍“去城”二字。（郭）

［3］开方式的现代形式为：$-2x+5000=0$。（陈）

［4］此应用刘徽提出的表去城公式：$l=\frac{b_1(b_2-D)}{D-b_1}$。（郭）

【今译】

今有正方形的城，不知其大小。树立两支表，东西相距$43\frac{2}{10}$步，使与人目相齐，以索连接之。令东表与城东南角、东北角三者共线。从东表退行$14\frac{8}{10}$步，遥望城西北角，自东端入索10步，又向北行，距表$64\frac{8}{10}$步，遥望城西北角，恰与西表端三者共线。问：城方、表去城各为多少？

答：城方6里340步，去表10里$85\frac{1}{5}$步。

术：设天元一为城高，以如积方法求其解。得到5000作为常数项，-2作为一次项系数。常数项作为被除数，一次项系数作为除数，除之，得到城方。 如果求表去城，以入索乘北行距表，以两表相距除之，得数作为影差。影差内减去东表退行，余数作为除数。又从北行距表内减影差，其余数乘东表退行，作为被除数，除之，就是表去城的远近。符合所问。

7.

【原文】

今有营，居山顶，岩底有泉，欲汲而不知其深。偃矩山上，令勾高四尺。从勾端望泉，入下股六尺。又设重矩于上，其矩间相去一丈六尺。更从勾端望泉，入上股五尺六寸。[1] 问：岩深几何？

ying cha. Subtract from the *ying cha* the distance of the observer's first position from the standard for the divisor. From the whole distance walked by the observer subtract the *ying cha*, and multiply the remainder by the observer's first distance from the standard.[4] This result gives the dividend.

[Notes]

[1] This is the sort of problem of *wang yi* (surveying the city) of Liu Hui's *Hai Dao Suan Jing* (Sea Island Mathematical Manual). Let a side of the city be a, the distance from the standard to the city l, *ying cha* D, the distance between the two standards d, the east standard backward b_1, the west standard backward b_2, and the line of sight cuts from the standard c. The formulas of *ying cha* and the side of the city given by Liu Hui are as follows:

$$D = \frac{cb_2}{d}, \quad a = \frac{c(b_2 - b_1)}{c - a}. \quad (G)$$

[2] *Qu biao qu cheng* is used in the original text. A redundant *qu* before *biao*, or a redundant *qu cheng* before *qu biao* is added. (G)

[3] The expression in modern form is the equation: $-2x + 5000 = 0$. (C)

[4] Liu Hui's formula for the distance from the standard to the city is used here: $l = \dfrac{b_1(b_2 - D)}{D - b_1}$. (G)

7. A camp is situated on the top of a hill. At the foot of a rocky cliff is a fountain from which they draw water. To find out the depth of the cliff they use two right triangles. The *gou*(short leg)of each being 4 *chi* arranged one above the other with a distance of 16 *chi* between their *gu*. As they look down from the vertex of the lower triangle to the fountain the line of vision cuts 6

答曰：岩深二十二丈。

术曰：立天元一为岩深，如积求之，得二十二尺为正实，一寸为从方。上实，下法而一，[2] 即岩深。合问。

【注释】

[1] 此问是刘徽《海岛算经》之望谷类问题。此岩深即谷深。记谷深、勾高、矩间、上股、下股分别为 h，a，d，b_1，b_2，刘徽提出谷深公式：$h=\dfrac{db_1}{b_2-b_1}-a$。（郭）

[2] 开方式的现代形式为：$x+220=0$。（陈）

【今译】

今有在山顶上有一营房，岩底下有泉水，欲汲取泉水而不知它的深。在山上放置一矩，令勾高4尺，从勾的上端望泉水，入下股6尺。再将此矩放置于上方，矩间相距1丈6尺，又从勾的上端望泉水，入上股5尺6寸。问：岩深为多少？

答：岩深22丈。

术：设天元一为岩深，以如积方法求其解。得到22尺作为常数项，1寸作为一次项系数。常数项作为被除数，一次项系数作为除数，除之，就是岩深。符合所问。

8.

【原文】

今有登山临邑，不知门高。偃矩山上，令勾高三尺。斜望门额，入下股四尺八寸。复望门阃，入下股二尺八寸八分。又复立重矩于上，其间相去五尺。更从勾端斜望门额，入上股三尺六寸。又望门阃，入上股二尺四寸。[1] 问：城门高几何？

答曰：门高一丈。

术曰：立天元一为门高，如积求之，得五十寸为正实，五分为益方，开无隅平方而一，[2] 得门高。合问。

chi from the gu. As they look down from the vertex of the upper triangle to the fountain the line of vision cuts 5 *chi* 6 *cun* from its *gu*. [1] How far below the first triangle is the cliff?

Ans. 220 *chi*.

Process. Let the element *tian* be the distance of the cliff below the triangle. From the statement we have 22 *chi* for the positive *shi* and 1 *cun* for the positive *fang*, a linear expression [2] whose root is the required distance.

[Notes]

[1] This is the sort of problem of *wang gu* (surveying the cliff) of Liu Hui's *Hai Dao Suan Jing* (Sea Island Mathematical Manual). The distance from the vertex of the upper triangle to the fountain is the depth of the cliff. Let the depth of the cliff *be* h, the height of *gou* a, the distance between the two right triangles d, the upper *gu* b_1, and the lower *gu* b_2. The formula of the depth of a cliff is as follows: $h = \dfrac{db_1}{b_2 - b_1} - a$. (G)

[2] The expression in modern form is the equation: $x + 220 = 0$. (C)

8. To measure the gate of a city, use two equal right triangles, one placed 5 *chi* above the other. The *gou* (short leg) of each triangle is 3 *chi*. The line of vision from the vertex of the lower triangle to the top of the gate cuts from its *gu* 4 *chi* 8 *cun*; the line of the vision to the bottom of the gate cuts from its *gu* 2 *chi* 8 *cun* 8 *fen*. The line of vision from the vertex of the upper triangle to the top of the gate cuts from its *gu* 3 *chi* 6 *cun*; the line of vision to the bottom of the gate cuts from its *gu* 2 *chi* 4 *cun*. [1] Find the height of the city gate.

Ans. Height of gate, 10 *chi*.

【注释】

[1] 此问是刘徽《海岛算经》之望清渊类问题。此之门高即水深，门额即水岸，门阃即白石。记水深、矩间、望岸上股、望岸下股、望石上股、望石下股分别为h, d, b_1, b_2, c_1, c_2, 刘徽提出水深公式：$h = \frac{d[c_1(b_2 - b_1) - b_1(c_2 - c_1)]}{(b_2 - b_1)(c_2 - c_1)}$。（郭）

[2] 开方式的现代形式为：$-5x + 5000 = 0$。（陈）

【今译】

今有登上山顶而望一座城，不知道城门的高。在山上放置一矩，令勾高3尺，从勾的上端斜望门额，入下股4尺8寸。再望门槛，入下股2尺8寸8分。又将此矩放置于上方，矩间相距5尺，再从勾的上端斜望门额，入上股3尺6寸。又望门槛，入上股2尺4寸。问：城门高为多少？

答：门高1丈。

术：设天元一为城门高，以如积方法求其解。得到50寸作为常数项，−5分作为一次项系数，开无隅平方，便得到城门高。符合所问。

Process. Let the element *tian* be the height of the city gate. From the statement we have 50 *cun* for the positive *shi* and 5 *fen* for the negative *fang*, a linear expression [2] whose root is the required height.

【 Notes 】

[1] This is the sort of problem of *wang qing yuan* of Liu Hui's *Hai Dao Suan Jing* (Sea Island Mathematical Manual). The height of the city gate is the depth of the water. The top of the gate is the bank of the river, and the bottom of the gate is the white stone. Let the depth of the water be h, the distance between the two right triangles d, the upper *gu* of *wang an* (surveying the bank) b_1, the lower *gu* of *wang an* b_2, the upper *gu* of *wang shi* (surveying the stone)c_1, and the lower *gu* of *wang shi* c_2. The formula of the depth of the water given by Liu Hui is as follows: $h = \dfrac{d[c_1(b_2 - b_1) - b_1(c_2 - c_1)]}{(b_2 - b_1)(c_2 - c_1)}$. (G)

[2] The expression in modern form is the equation: $-5x + 5000 = 0$. (C)

或问歌彖　一十二问

1.

【原文】

或问：今有方池一所，每面丈四方停。葭生西岸长其形，出水三十寸整。东岸蒲生一种，水上一尺无零。葭、蒲稍接水齐平，[1]借问：三般怎定？

答曰：水深一丈二尺，蒲长一丈三尺，葭长一丈五尺。

术曰：立天元一为水深，如积求之，得二千一百六十为正实，一百九十二为益方，一为正隅，平方开之，[2]合问。又立天元一为蒲长，如积求之，得二千三百五十三为正实，一百九十四为益方，一为正隅，平方开之，[3]合问。又立天元一为葭长，如积求之，得二千七百四十五为正实，一百九十八为益方，一为从隅，平方开之。[4]合问。

【注释】

[1]蒲、葭稍相接处将水面分成两部分，分别是两个皆以水深b为股的勾股形的勾，记为a_1，a_2，它们的弦分别是$b+3$，$b+1$，于是$a_1^2+b^2=(b+3)^2$，$a_2^2+b^2=(b+1)^2$，$a_1+a_2=14$。这是由勾a和股弦差$c-b$求股b，弦c的问题，《九章算术》勾股章使用了公式$b=\dfrac{a^2-(c-b)^2}{2(c-b)}$，$c=\dfrac{a^2+(c-b)^2}{2(c-b)}$。（郭）

[2]开方式的现代形式为：$x^2-192x+2160=0$。（陈）

[3]开方式的现代形式为：$x^2-194x+2353=0$。（陈）

[4]开方式的现代形式为：$x^2-198x+2745=0$。（陈）

【今译】

有人问：假设方池一所，每边1丈4尺。一株芦苇生西岸，出水恰好3尺。一棵蒲草生东岸，水上刚好1尺。芦、蒲稍接与水齐，请问：三事为多少？

Huo Wen Ge Tuan (Problems in Poetic Form)

12 Problems

1. A side of a square pool is 14 *chi*. A reed growing on the west edge extends 30 *cun* above the water. A rush growing on the east edge extends 1 *chi* above the water. If the tops of the two are tied together they will meet on a level with the water. [1] Now tell me, how we can decide the three questions.

Ans. Depth of the water, 1 *zhang* 2 *chi*;

length of the reed, 1 *zhang* 3 *chi*;

length of the rush, 1 *zhang* 5 *chi*.

Process. Let the element *tian* be the depth of the water. From the statement we have 2160 for the positive *shi*, 192 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression [2] whose root is the required depth.

Again let the element *tian* be the length of the rush. From the statement we have 2353 for the positive *shi*, 194 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression [3] whose root is the length of the rush.

Again let the element *tian* be the length of the reed. From the statement we have 2745 for the positive *shi*, 198 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression [4] whose root is the length of the reed.

[Notes]

[1] Where the reed and the rush meet divides the water into two parts. They are two *gou* of two right triangles. They both equal to the depth of the water b. Let they be a_1 and a_2.

答：水深1丈2尺，蒲长1丈3尺，葭长1丈5尺。

术：设天元一为水深，以如积方法求其解。得到2160作为常数项，−192为一次项系数，1为最高次项系数，开平方，即得。符合所问。又：设天元一为蒲长，以如积方法求其解。得到2353作为常数项，−194为一次项系数，1为最高次项系数，开平方，即得。符合所问。又：设天元一为葭长，以如积方法求其解。得到2745作为常数项，−198为一次项系数，1为最高次项系数，开平方，即得。符合所问。

2.

【原文】

或问：务前听得语云云：新熟醇、醨共一盆，醇酒一升醉三客，醨酒三升醉一人。都来共饮十二斗，座中醉倒五十人。[1]借问四方能算者，几多醨酒几多醇？

答曰：醇酒三升七合半，醉一十一人四分人之一；

醨酒一硕一斗六升二合半，醉三十八人四分人之三。

术曰：立天元一为醇酒数，如积求之，得三十为益实，八为从方。上实，下法而一，[2]得醇酒数。又：立天元一为醨酒数，如积求之，得九百三十为正实，八为益方，开无隅平方而一，[3]得醨酒。又：立天元一为饮醇酒人数，如积求之，得九十为正实，八为益方。上实，下法而一，[4]得醇酒人。又：立天元一为饮醨酒人数，如积求之，得三百一十为益实，八为从方，开无隅平方而一，[5]得醨酒人数。不尽者，约之。合问。

Their *xian* are $b+3$ and $b+1$, respectively. Then, $a_1^2+b^2=(b+3)^2$, $a_2^2+b^2=(b+1)^2$, $a_1+a_2=14$. This is a problem to find *gu* b and *xian* c from *gou* a and *gu xian cha* $c-b$. The chapter *gou gu* of *The Nine Chapters of Mathematical Procedures* used the following formulas: $b=\dfrac{a^2-(c-b)^2}{2(c-b)}$, $c=\dfrac{a^2+(c-b)^2}{2(c-b)}$. (G)

[2] The expression in modern form is the equation: $x^2-192x+2160=0$. (C)

[3] The expression in modern form is the equation: $x^2-194x+2353=0$. (C)

[4] The expression in modern form is the equation: $x^2-198x+2745=0$. (C)

2. I heard some one talking in the midst of the fog saying he had a mixture of wine, weak and strong; also that three persons could be intoxicated by means of the strong while three *sheng* of the weak could intoxicate only one.Fifty persons here were found lying in their seats after twelve *dou* were gone.[1] May I ask those who are skilled in calculation, those who come from the four directions, How much was in the mixture, the weak and the strong?

Ans. Strong, 3 *sheng* $7\frac{1}{2}$ *he*, intoxicating $11\frac{1}{4}$ persons;

weak, 1 *shuo* 1 *dou* 6 *sheng* $2\frac{1}{2}$ *he*, intoxicating $38\frac{3}{4}$ persons.

Process. Let the element *tian* be the amount of the strong wine. From the statement we have 30 for the negative *shi* and 8 for the positive *fang*, a linear expression[2] whose root is the required amount of the strong wine. Again let the element *tian* be the amount of the weak wine. From the statement we have 930 for the positive *shi* and 8 for the negative *fang*, a linear expression[3] whose root is the required amount of the weak wine. Again

【注释】

[1] 务，店铺，宋元俗语指酒店。醨，薄酒。记醇酒与醉倒之人、醨酒与醉倒之人分别为 a，m，b，n，此给出关系式：$3a = m$，$\frac{1}{3}b = n$，$a + b = 12$，$m + n = 50$。（郭）

[2] 开方式的现代形式为：$8x - 30 = 0$。（陈）

[3] 开方式的现代形式为：$-8x + 930 = 0$。（陈）

[4] 开方式的现代形式为：$-8x + 90 = 0$。（陈）

[5] 开方式的现代形式为：$8x - 310 = 0$。（陈）

【今译】

有人问：酒店门前听人云：新酿醇、醨共一盆。醇酒一升醉3客，醨酒三升醉一人。总共豪饮12斗，座中醉倒50人。借问四方能算者，几多醨酒几多醇？

答：醇酒3升$7\frac{1}{2}$合，醉倒$11\frac{1}{4}$人；

醨酒1石1斗6升$2\frac{1}{2}$合，醉倒$38\frac{3}{4}$人。

术：设天元一为醇酒数，以如积方法求其解。得到-30作为常数项，8作为一次项系数。常数项作为被除数，一次项系数作为除数，除之，得醇酒数。 又：设天元一为醨酒数，以如积方法求其解。得到930作为常数项，-8作为一次项系数。开无隅平方除之，得醨酒数。 又：设天元一为饮醇酒人数，以如积方法求其解。得到90作为常数项，-8作为一次项系数。常数项作为被除数，一次项系数作为除数，除之，得饮醇酒人数。 又：设天元一为饮醨酒人数，以如积方法求其解。得到-310作为常数项，8作为一次项系数。开无隅平方除之，得饮醨酒人数。除之不尽，约分。符合所问。

let the element *tian* be the number of persons who were intoxicated by means of the strong wine. From the statement we have 90 for the positive *shi* and 8 for the negative *fang*, a linear expression [4] whose root is the required number of the persons. Again let the element *tian* be the number of the persons who were intoxicated by means of the weak wine. From the statement we have 310 for the negative *shi* and 8 for the positive *fang*, a linear expression [5] whose root is the required number.

【Notes】

[1] The *wu* is a drinkery in the saying of Song and Yuan dynasties. The *li* refers to the weak wine. Let the quantity of the strong wine and the number of the intoxicated persons be a and m, the quantity of the weak wine and the number of the intoxicated persons be b and n. From the statement, we have the following expression: $3a = m$, $\frac{1}{3}b = n$, $a + b = 12$, $m + n = 50$. (G)

[2] The expression in modern form is the equation: $8x - 30 = 0$. (C)

[3] The expression in modern form is the equation: $-8x + 930 = 0$. (C)

[4] The expression in modern form is the equation: $-8x + 90 = 0$. (C)

[5] The expression in modern form is the equation: $8x - 310 = 0$. (C)

3.

【原文】

或问：今有直田一亩足，正向中间生竿竹。四角至竹各十三，[1] 借问四事元数目。

答曰：长二十四步，阔一十步；和三十四步，较一十四步[2]。

术曰：立天元一为长，如积求之，得五万七千六百为正实，六百七十六为益上廉，一为正隅，三乘方开之，[3] 得长。又：立天元一为阔，如积求之，得五万七千六百为益实，六百七十六为从上廉，一为益隅，三乘方开之，[4] 得阔。又：立天元一为和，如积求之，得一千一百五十六为益实，一为正隅，平方开之[5]，得和。又：立天元一为较，如积求之，得一百九十六为正实，一为负隅，平方开之，[6] 得较。合问。

【注释】

[1] 此即：$ab = 240$，$a^2 + b^2 = (2 \times 13)^2$。（郭）

[2] 从上问“四事”，下有求和、较的术文，此处脱和、较的答案，今补。（郭）

[3] 开方式的现代形式为：$x^4 - 676x^2 + 57600 = 0$。（陈）

[4] 开方式的现代形式为：$-x^4 + 676x^2 - 57600 = 0$。（陈）

[5] 开方式的现代形式为：$x^2 - 1156 = 0$。（陈）

[6] 开方式的现代形式为：$-x^2 + 196 = 0$。（陈）

【今译】

有人问：假设直田整1亩，正中生长一株竹。竹至四角各13，借问四事原数目。

答：长24步，阔10步；长阔和34步，长阔差14步。

术：设天元一为直田的长，以如积方法求其解。得到57600作为常数项，−676作为二次项系数，1作为最高次项系数，开四次方，得到长。

3. In the middle of a *mu* of land in the form of a rectangle grows a bamboo. The distance of the bamboo from each vertex is 13 (*bu*). [1] Tell me, please, what are the numbers for the four dimensions?

Ans. Length, 24 *bu*; width, 10 *bu*; *he*, 34 *bu*; *jiao*, 14 *bu* [2].

Process. Let the element *tian* be the length of the rectangle. From the statement we have 57600 for the positive *shi*, 676 for the negative upper *lian*, and 1 for the positive *yu*, a biquadratic expression [3] whose root is the length of the rectangle. Again let the element *tian* be the width of the rectangle. From the statement we have 57600 for the negative *shi*, 676 for the positive upper *lian*, and 1 for the negative *yu*, a biquadratic expression [4] whose root is the width of the rectangle. Again let the element *tian* be the *he*. From the statement we have 1156 for the negative *shi*, and 1 for the positive *yu*, a quadratic expression [5] whose root is the required *he*. Again let the element *tian* be the *jiao*. From the statement we have 196 for the positive *shi*, and 1 for the negative *yu*, a quadratic expression [6] whose root is the required *jiao*.

【Notes】

[1] That is, $ab = 240, a^2 + b^2 = (2 \times 13)^2$. (G)

[2] Because there are "four dimensions" before the answer and the process for getting the *he* and the *jiao* behind it, the answers for the *he* and the *jiao* are lost here. Now we add them. (G)

[3] The expression in modern form is the equation: $x^4 - 676x^2 + 57600 = 0$. (C)

又：设天元一为直田的阔，以如积方法求其解。得到 −57600 作为常数项，676 作为二次项系数，−1 作为最高次项系数，开四次方，得到阔。　又：设天元一为长阔和，以如积方法求其解。得到 −1156 作为常数项，1 作为最高次项系数，开平方，得到长阔和。　又：设天元一为长阔差，以如积方法求其解。得到 196 作为常数项，−1 作为最高次项系数，开平方，得到长阔差。符合所问。

4.

【原文】

或问：我有一壶酒，携著游春走。遇务添一倍，逢店饮斗九。店务经四处，没了壶中酒。[1] 借问此壶中，当元多少酒。

答曰：一斗七升八合一勺二抄五撮。

术曰：立天元一为当元壶中酒，如积求之，得二百八十五为益实，一十六为从方。上实，下法而一，[2] 合问。

【注释】

[1] 此即：$2\{2[2(2x-19)-19]-19\}-19=0$。（郭）

[2] 开方式的现代形式为：$16x-285=0$。（陈）

【今译】

有人问：我有一壶酒，携着去春游。遇到酒坊添一倍，逢酒店饮 1 斗 9。酒坊、酒店各四处，喝完壶中酒。借问此壶中，原有多少酒。

答：1 斗 7 升 8 合 1 勺 2 抄 5 撮。

术：设天元一为壶中原有酒数，以如积方法求其解。得到 −285 作为常数项，16 作为一次项系数。常数项作为被除数，一次项系数作为除数，除之，即得。符合所问。

[4] The expression in modern form is the equation: $-x^4 + 676x^2 - 57600 = 0$. (C)

[5] The expression in modern form is the equation: $x^2 - 1156 = 0$. (C)

[6] The expression in modern form is the equation: $-x^2 + 196 = 0$. (C)

4. I had some wine in a bottle which I took with me everywhere I went in the lovely spring weather. Whenever I met a drinkery I added to the wine as much as was left in the bottle. Every time I stopped at a good place I drank one *dou* and nine *sheng*. Four times I met drinkeries and four times I stopped at good places. I found my bottle was entirely empty. [1] Find out for me, please, how much wine there was when I started.

Ans. 1 *dou* 7 *sheng* 8 *he* 1 *shao* 2 *chao* 5 *cuo*.

Process. Let the element *tian* be the amount of the wine in the bottle when he started from home. From the statement we have 285 for the negative *shi*, and 16 for the positive *fang*, a linear expression [2] whose root is the required amount of the wine.

[Notes]

[1] That is, $2\{2[2(2x - 19) - 19] - 19\} - 19 = 0$. (G)

[2] The expression in modern form is the equation: $16x - 285 = 0$. (C)

5.

【原文】

或问：九百九十九文钱，及时梨、果买一千。一十一文梨九个，七枚果子四文钱。[1]

答曰：梨六百五十七个，价八百三文；

果三百四十三枚，价一百九十六文。

术曰：立天元一为梨数，如积求之，得二万六千九百三十七为益实，四十一为从方，开无隅平方而一，[2] 得梨。 又：立天元一为果数，如积求之，得一万四千六十三为正实，四十一为益方，上实，下法而一，[3] 得果。 又：立天元一为梨价，如积求之，得三万二千九百二十三为正实，四十一为益方，开无隅平方除之，[4] 得梨价。 又：立天元一为果价，如积求之，得八千三十六为正实，四十一为益方，上实，下法而一，[5] 得果价。合问。

【注释】

[1] 记梨数及其价，果数及其价分别为 a，m，b，n，此给出关系式：$\frac{11}{9}a = m$，$\frac{4}{7}b = n$，$\frac{11}{9}a + \frac{4}{7}b = 999$，$a + b = 1000$。（郭）

[2] 开方式的现代形式为：$41x - 26937 = 0$。（陈）

[3] 开方式的现代形式为：$-41x + 14063 = 0$。（陈）

[4] 开方式的现代形式为：$-41x + 32923 = 0$。（陈）

[5] 开方式的现代形式为：$-41x + 8036 = 0$。（陈）

【今译】

有人问：我有999文钱，买时鲜梨、果1000。11文钱买9梨，7枚果子4文钱。借问四方能算者，几多梨、果几文钱？

5. With 999 cash I bought 1000 pears and apples. I bought the pears at the rate of 9 for 11 cash and the apples at the rate of 7 for 4 cash.[1] How many of each did I buy?

Ans. Number of pears, 657;

cost of pears, 803 cash;

number of apples, 343;

cost of apples, 196 cash.

Process. Let the element *tian* be the number of the pears. From the statement we have 26937 for the negative *shi*, and 41 for the positive *fang*, a linear expression[2] whose root is the number of the pears. Again let the element *tian* be the number of the apples. From the statement we have 14063 for the positive *shi*, and 41 for the negative *fang*, a linear expression[3] whose root is the number of the apples. Again let the element *tian* be the number of the cash spent for the pears. From the statement we have 32923 for the positive *shi*, and 41 for the negative *fang*, a linear expression[4] whose root is the number of the cash spent for the pears. Again let the element *tian* be the number of the cash spent for the apples. From the statement we have 8036 for the positive *shi*, and 41 for the negative *fang*, a linear expression[5] whose root is the number of the cash spent for the apples.

【Notes】

[1] Let the number of the pears and its price be a and m, and the number of the apples and its price b and n. The expression is as follows:

$$\frac{11}{9}a = m, \frac{4}{7}b = n, \frac{11}{9}a + \frac{4}{7}b = 999, a + b = 1000. \text{ (G)}$$

答：梨657个，价钱803文；

果子343枚，价钱196文。

术：设天元一为梨数，以如积方法求其解。得到−26937作为常数项，41作为一次项系数，开无隅平方除之，得梨数。 又：设天元一为果数，以如积方法求其解。得到14063作为常数项，−41作为一次项系数。常数项作为被除数，一次项系数作为除数，除之，得果数。 又：设天元一为梨价，以如积方法求其解。得到32923作为常数项，−41作为一次项系数，开无隅平方除之，得梨价。 又：设天元一为果价，以如积方法求其解。得到8036作为常数项，−41作为一次项系数，常数项作为被除数，一次项系数作为除数，除之，得果价。符合所问。

6.

【原文】

或问：院内鞦韆跳起，杆、索未审高低。脚登画版女娇嬉，离地版高一尺。只见送行两步，版高三尺无奇。[1] 杆、绳长短怎生知，除演天元如积。

答曰：杆长二丈七尺，索长二丈六尺。

术曰：立天元一为杆长，如积求之，得一百八为正实，四为益方，开无隅平方除之，[2] 得杆长。 又：立天元一为索长，如积求之，得一百四为正实，四为益方，上实，下法而一，[3] 得索长。合问。

【注释】

[1] 记杆长、索长分别为l，c，则形成一个以2步即10尺为勾，$l-3$为股，索长c为弦的勾股形，有关系式：$l-1=c$，于是$10^2+(l-3)^2=(l-1)^2$。(郭)

[2] 开方式的现代形式为：$-4x+108=0$。(陈)

[2] The expression in modern form is the equation: $41x - 26937 = 0$. (C)

[3] The expression in modern form is the equation: $-41x + 14063 = 0$. (C)

[4] The expression in modern form is the equation: $-41x + 32923 = 0$ (C)

[5] The expression in modern form is the equation: $-41x + 8036 = 0$. (C)

6. There is a swing in my yard. The height of the pole and the length of the rope are unknown. The feet of my pretty little maid were on the board which is one *chi* from the ground. Pushing her two steps forward I found that the board was just three *chi* from the ground. [1] How can the height of the pole and the length of the rope be known without the knowledge of solving the expression with the element *tian* as unknown?

Ans. Length of the pole, 27 *chi*;

length of the rope, 26 *chi*.

Process. Let the element *tian* be the length of the pole. From the statement we have 108 for the positive *shi* and 4 for the negative *fang*, a linear expression [2] whose root is the length of the pole. Again let the element *tian* be the length of the rope. From the statement we have 104 for the

[3] 开方式的现代形式为：$-4x+104=0$。（陈）

【今译】

有人问：院内秋千荡起，杆、索长短不知。女郎嬉蹬画版，版高离地1尺。只见送行2步，版高恰恰3尺。杆索长短怎么知，除演天元、如积。

答：杆长2丈7尺，索长2丈6尺。

术：设天元一为杆长，以如积方法求其解。得到108作为常数项，−4作为一次项系数，开无隅平方除之，得杆长。　又：设天元一为索长，以如积方法求其解。得到104作为常数项，−4作为一次项系数。常数项作为被除数，一次项系数作为除数，除之，得索长。符合所问。

7.

【原文】

或问：六贯二百一十钱，倩人去买几株椽。每株脚钱三文足，无钱准与一株椽。[1]

答曰：椽四十六株，株价一百三十五文。

术曰：立天元一为椽数，如积求之，得二千七十为益实，一为益方，一为从隅，平方开之，[2] 得椽数。　又：立天元一为每株椽价，如积求之，得一万八千六百三十为益实，三为从方，一为正隅，平方开之，[3] 得株价。合问。

【注释】

[1] 记椽数及每株价分别为a，m，此给出关系式：$am=6210$，$3(a-1)=m$。（郭）

positive *shi* and 4 for the negative *fang*, a linear expression[3] whose root is the length of the rope.

[Notes]

[1] Let the length of the pole and the length of the rope be l and c. Then, a right triangle is formed with its *gou* 2 *bu*, that is, 10 *chi*, its *gu* $l-3$, and its *xian* c. And we know, $l-1=c$, then, $10^2+(l-3)^2=(l-1)^2$. (G)

[2] The expression in modern form is the equation: $-4x+108=0$. (C)

[3] The expression in modern form is the equation: $-4x+104=0$. (C)

7. I asked some one to buy me some beams for 6210 cash. The cost of transportation was three per beam which was paid by giving one beam to the transporter.[1]

Ans. Number of beams, 46;

price per beam, 135 cash.

Process. Let the element *tian* be the number of the beams. From the statement we have 2070 for the negative *shi*, 1 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[2] whose root is the number of the beams.

Again let the element *tian* be the price per beam. From the statement we have 18630 for the negative *shi*, 3 for the positive *fang*, and 1 for the positive *yu*, a quadratic expression[3] whose root is the price per beam.

[Notes]

[1] Let the number of the beams and the price per beam be a and m. The expression

［2］开方式的现代形式为：$x^2-x-2070=0$。（陈）

［3］开方式的现代形式为：$x^2+3x-18630=0$。（陈）

【今译】

有人问：6贯210钱，请人去买几株椽。每株脚钱3文整，无钱准折1株椽。借问四方能算者，多少椽子株几钱。

答：椽子46株，每株价135文。

术：设天元一为为椽子数，以如积方法求其解。得到−2070作为常数项，−1为一次项系数，1为最高次项系数，开平方，便得到椽子数。

又：设天元一为每株椽子的单价，以如积方法求其解。得到−18630作为常数项，3为一次项系数，1为最高次项系数，开平方，便得到每株单价。符合所问。

8.

【原文】

或问：方城里周六十四，假使金砖遍铺地。每条均铸厚一寸，长、阔相和恰一尺。寸金十五两为法，尚带零铢一十八。每砖计重十七觔、一十五两六铢答、七丝二黍在其中，共是一砖之重率。[1] 长阔金砖用几何，恼得先生没乱杀。

答曰：阔二寸四分，长七寸六分；

砖四十五亿四千七百三十六万八千四百二十一枚一十九分枚之一；

重八百一十六亿四千八百万觔。

术曰：立天元一为砖阔，如积求之，得一十八寸二分四厘为正实，一十寸为益方，一寸为正隅，平方开之，[2] 得砖阔。 求砖数者，以寸里法通城积为实，以一砖之积寸为法，实如法而一。不尽，约之，为分。[3] 合问。

is given as follows: $am = 6210, 3(a-1) = m$. (G)

[2] The expression in modern form is the equation: $x^2 - x - 2070 = 0$. (C)

[3] The expression in modern form is the equation: $x^2 + 3x - 18630 = 0$. (C)

8. The perimeter of a square city is 64 *li*. The ground of the city is to be covered with gold bricks one *cun* in thickness, the sum of the other two dimensions being one *chi*. One *cun* of gold weights 15 *liang* 18 *zhu*, and each brick weighs 17 *jin* 15 *liang* 6 *zhu* 7 *si* 2 *shu*. [1] To find how many gold bricks will be needed makes the master confused.

Ans. Width, 2 *cun* 4 *fen*; length, 7 *cun* 6 *fen*;
number of the bricks, $4547368421\frac{1}{19}$;
weight, 81648000000 *jin*.

Process. Let the element *tian* be the width of a brick. From the statement we have 18 *cun* 2 *fen* 4 *li* for the positive *shi*, 10 *cun* for the negative *fang*, and 1 *cun* for the positive *yu*, a quadratic expression [2] whose root is the width of a brick. To find the number of a brick, divide the area of the city by the area of a brick, and the quotient will be the number of the bricks needed. If there is a remainder, express it as a fraction in lowest terms [3].

【注释】

[1] 记方城的每边长为A，则$A=\frac{64}{4}=16$，其面积为16^2里 = 82944000000寸。金砖的阔、长分别为a, b，则$a+b=10$，其面积为ab，其重量为15两18铢×（$ab\times 1$）= 17斤15两6铢7丝2黍。（郭）

[2] 开方式的现代形式为：$x^2-10x+18.24=0$。（陈）

[3] 砖数 = $\frac{82944000000}{2.4\times 7.6}$。（郭）

【今译】

有人问：方城周长64里，假使金砖遍铺地。每条金砖厚1寸，长阔相加恰1尺。1寸金重15两，又18铢为零奇。每条砖重17斤、15两6铢7丝2黍。金砖长阔各多少，金砖共用多少枚?

答：金砖阔2寸4分，长7寸6分。

金砖数$4547368421\frac{1}{19}$枚。

金砖总重81648000000斤。

术：设天元一为金砖之阔，以如积方法求其解。得到18.24作为常数项，−10为一次项系数，1为最高次项系数，开平方，便得到金砖之阔。 求砖数的方法：以1平方里含的平方寸数乘方城的面积作为被除数，以1砖的面积作为除数，除之，即得。除不尽，约简，作为分数部分。符合所问。

9.

【原文】

或问：今有人来赎解，本多利少难评。共收四贯别无零，说破源流即省。本利各开方毕，并之与日相停。若还相减甚分明，四十文差余剩。[1]

答曰：本钱三贯六百文，月利四十一文三分文之二；

【Notes】

[1] Let every side of the square city be A. Then, $A = \frac{64}{4} = 16$. Its area is as follows: 16^2 *li* = 82944000000 *cun*. The width and the length of the gold brick are a and b, then $a + b = 10$. Its area is ab. Its weight is as follows: 15 liang 18 zhu × (ab × 1) = 17 *jin* 15 *liang* 6 *zhu* 7 *si* 2 *shu*. (G)

[2] The expression in modern form is the equation: $x^2 - 10x + 18.24 = 0$. (C)

[3] The number of the bricks is $\frac{82944000000}{2.4 \times 7.6}$. (G)

9. The principal and interest for a certain length of time amounts to 4000 cash. The sum of their roots equals the number of days; the difference between their square roots is 40.[1]

Ans. Principal, 3600 cash;

interest for one month, $41\frac{2}{3}$ cash;

两个月二十日，利钱四百文。

术曰：立天元一为本钱，地元一为利钱，天、地配合求之，得一百四十四万为益实，四千为从方，一为益隅，平方开之，得本钱。[2] 余依加减求之。合问。

【注释】

[1] 记本钱、利钱分别为 a，b，则 $a+b=4000$。记日数为 m，则 $\sqrt{a}+\sqrt{b}=m$，$\sqrt{a}-\sqrt{b}=40$。（郭）

[2] 开方式的现代形式为：$-x^2+4000x-1440000=0$。（陈）

【今译】

有人问：有人来赎典当，本多利少难评。共收本利4贯整，说破源流悟省。本利各自开方，相加与日数相等。若还相减甚分明，相差40文无剩。本利日数怎么知，演二元、如积君请。

答：本钱3贯600文，月利 $41\frac{2}{3}$ 文；

2月20日，利钱400文。

术：设天元一为本钱，地元一为利钱。天元与地元配合求之，得到 -1440000 为常数项，4000为一次项系数，-1 为最高次项系数，开平方，得到本钱。其余各数依照加减求之。符合所问。

10.

【原文】

或问：元有直田一亩地，横行六步竖行四。斜行十五至隅头，[1] 借问长平数目事。

答曰：长一十六步，阔一十五步。

number of days, 2 months 20 days;

total interest, 400 cash.

Process. Let the element *tian* be the principal, and the element *di* the interest. From the statement we have 1440000 for the negative *shi*, 4000 for the positive *fang*, and 1 for the negative *yu*, a quadratic expression [2] whose root is the required principal.

[Notes]

[1] Let the principal and the interest be a and b. Then, $a + b = 4000$. Let the number of the days be m, then $\sqrt{a} + \sqrt{b} = m$, $\sqrt{a} - \sqrt{b} = 40$. (G)

[2] The expression in modern form is the equation: $-x^2 + 4000x - 1440000 = 0$. (C)

10. There is a mu of land in the form of a rectangle. If one walks from one corner to the diagonal corner going horizontally 6 *bu*, vertically 4 *bu*, and then diagonally 15 *bu*, he will reach the corner of the land. [1] Find the dimensions of the land.

Ans. Length 16 *bu*; width, 15 *bu*.

术曰：立天元一为长，地元一为平，天、地配合求之，得五万七千六百为正实，二千八百八十为益方，一百七十三为益上廉，八为益下廉，一为正隅，三乘方开之，[2] 得长。 又：立天元一为阔，地元一为长，天、地配合求之，得五万七千六百为正实，一千九百二十为益方，一百七十三为益上廉，一十二为益下廉，一为正隅，三乘方开之，[3] 得阔。合问。

【注释】

[1] 记直田的阔、长分别为 a，b，则 $ab=240$，$(a-6)^2+(b-4)^2=15^2$。（郭）

[2] 开方式的现代形式为：$x^4-8x^3-173x^2-2880x+57600=0$。（陈）

[3] 开方式的现代形式为：$x^4-12x^3-173x^2-1920x+57600=0$。（陈）

【今译】

有人问：原有直田1亩地，横行6步竖行4，斜行15至隅角，借问长阔各多少。

答：直田的长16步，阔15步。

术：设天元一为直田的长，地元一为阔。天元与地元配合求之，得到57600为常数项，−2880为一次项系数，−173为二次项系数，−8为三次项系数，1为最高次项系数，开四次方，得到长。 又：设天元一为直田的阔，地元一为长。天元与地元配合求之，得到57600为常数项，−1920为一次项系数，−173为二次项系数，−12为三次项系数，1为最高次项系数，开四次方，得到阔。符合所问。

Process. Let the element *tian* be the length of the land and the element *di* the width. From the statement we have 57600 for the positive *shi*, 2880 for the negative *fang*, 173 for the negative upper *lian*, 8 for the negative lower *lian*, and 1 for the positive *yu*, an expression[2] of the fourth degree whose root is the required length. Again let the element *tian* be the width of the land and the element *di* the length. From the statement we have 57600 for the positive *shi*, 1920 for the negative *fang*, 173 for the negative upper *lian*, 12 for the negative lower *lian*, and 1 for the positive *yu*, an expression[3] of the fourth degree whose root is the required width.

【Notes】

[1] Let the width and the length of the rectangle be a and b. Then, $ab = 240$, $(a - 6)^2 + (b - 4)^2 = 15^2$. (G)

[2] The expression in modern form is the equation: $x^4 - 8x^3 - 173x^2 - 2880x + 57600 = 0$. (C)

[3] The expression in modern form is the equation: $x^4 - 12x^3 - 173x^2 - 1920x + 57600 = 0$. (C)

11.

【原文】

或问：一只银盘三尺周，内容三只水晶球。若人算得穿心径，[1] 万两黄金也合酬。

答曰：五寸六十九分寸之二十五。

术曰：立天元一为球子径，如积求之，得三百为益实，六十为从方，一为正隅，平方开之，[2] 得球子径四寸。不尽，命分。以减盘径。合问。

【注释】

[1] 记球子径为 r，朱世杰认为：$(10-r)^2=r^2+\left(\frac{10+r}{2}\right)^2$。（郭）

[2] 开方式的现代形式为：$x^2+60x-300=0$。（陈）

【今译】

有人问：一只银盘3尺周，内容三只水晶球。若人算得穿心径，万两黄金也该酬。

答：穿心径为 $5\frac{25}{69}$ 寸。

术：设天元一为球径，以如积方法求其解。得到 -300 作为常数项，60作为一次项系数，1作为最高次项系数，开平方，得到球径4寸。开方不尽，以余开方式的实、方与隅命名其分数部分。减盘径，即得穿心径。符合所问。

12.

【原文】

或问：积减弦长与半平，余与三勾五股停。勾弦、股弦差相并，要作元

11. Three crystal balls lie on a silver plate whose circumference is 3 *chi*. If any one can find the segment of the diameter of the plate not covered by a ball he ought to be rewarded by 10000 taels of gold [1].

Ans. $5\frac{25}{69}$ *cun*.

Process. Let the element *tian* be the diameter of a ball. From the statement we have 300 for the negative *shi*, 60 for the positive *fang*, and 1 for the positive *yu*, a quadratic expression[2] whose root, $4\frac{44}{69}$ *cun*, is the diameter of a ball. Subtract this value from 1 *chi*, the diameter of the plate. The remainder, $5\frac{25}{69}$ *cun*, is the required segment.

[Notes]

[1] Let the diameter of the crystal ball be r. Zhu Shijie considered, $(10 - r)^2 = r^2 + (\frac{10 + r}{2})^2$. (G)

[2] The expression in modern form is the equation: $x^2 + 60x - 300 = 0$. (C)

12. The product of the *gou* and the *gu* minus the sum of the *xian* and one-half of the *gou* is equal to the sum of 3 times the *gou* and 5 times the *gu*; the sum of the *xian* minus the *gou* and the *xian* minus the *gu* is less than the *gu* by one-

长少半平。[1]

答曰：勾八步，股一十五步，弦一十七步。

术曰：立天元一为勾，地元一为股，人元一为弦。三才相配求之，得四百八十为益实，六十为从方，开无隅平方而一，[2] 得勾。开地元股：得四百八十为益实，三十二为从方，上实，下法除之，[3] 得股。

开人元弦：得五百一十为益实，三十为从方，开无隅平方除之，[4] 得弦。合问。

【注释】

[1] 此即：$ab-(c+\frac{1}{2}a)=3a+5b$，$(c-a)+(c-b)=b-\frac{1}{2}a$。（郭）

[2] 开方式的现代形式为：$60x-480=0$。（陈）

[3] 开方式的现代形式为：$32y-480=0$。（陈）

[4] 开方式的现代形式为：$30z-510=0$。（陈）

【今译】

有人问：面积减弦与半勾，余与3勾5股等。勾弦、股弦差相加，要作股长少半平。借问四方能算者，勾、股、弦长怎生定？

答：勾8步，股15步，弦17步。

术：设天元一为勾，地元一为股，人元一为弦。天元、地元与人元配合求之，得到−480为常数项，60为一次项系数，开无隅平方，得到勾。开地元股：得到−480为常数项，32为一次项系数。常数项作为被除数，一次项系数作为除数，除之，得到股。开人元弦：得到−510为常数项，30为一次项系数，开无隅平方，得到弦。符合问题。

half of the gou.[1]

Ans. *Gou*, 8 *bu*; *gu*, 15 *bu*; *xian*, 17 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the *xian*. From the statement we have 480 for the negative *shi*, and 60 for the positive *fang*, a linear expression [2] whose root is the required *gou*. 480 for the negative *shi* and 32 for the positive *fang*, a linear expression [3] whose root is the required *gu*. 510 for the negative *shi* and 30 for the positive *fang*, a linear expression [4] whose root is the required *xian*.

[Notes]

[1] That is, $ab - (c + \frac{1}{2}a) = 3a + 5b$, $(c - a) + (c - b) = b - \frac{1}{2}a$. (G)

[2] The expression in modern form is the equation: $60x - 480 = 0$. (C)

[3] The expression in modern form is the equation: $32y - 480 = 0$. (C)

[4] The expression in modern form is the equation: $30z - 510 = 0$. (C)

茭草形段　七问

1.

【原文】

今有茭草六百八十束，欲令落一形埵之。[1] 问：底子几何？

答曰：一十五束。

术曰：立天元一为落一底子，如积求之。得四千八十为益实，二为从方，三为从廉，一为正隅，立方开之。[2] 合问。

【注释】

[1]李善兰在其《则古昔斋算学》中称三角落一形垛为一个三次的三角垛。公式为：$S=\dfrac{n(n+1)(n+2)}{2\cdot 3}$。（陈）$n$层茭草形垛的求积公式是：$\sum_{i=1}^{n} i=\frac{1}{2}n(n+1)$。落一形垛又称为三角垛。$n$层落一形垛的求积公式是$S=\sum_{i=1}^{n}\frac{1}{2}i(i+1)=\frac{1}{3!}n(n+1)(n+2)$。据题意，$\frac{1}{3!}n(n+1)(n+2)=680$。（郭）

[2]开方式的现代形式为：$x^3+3x^2+2x-4080=0$。（陈）

【今译】

今有680束茭草，欲垛成落一形垛。问：其底子为多少束？

答：15束。

术：设天元一为落一形垛的底子，以如积方法求其解。得到-4080为常数项，2为一次项系数，3为二次项系数，1为最高次项系数，开立方，即得。符合所问。

Jiao Cao Xing Duan (Piles of Hay)
7 Problems

1. 680 bundles of hay are in the *luo yi* form.[1] Find the number of bundles in a side of the base.

Ans. 15 bundles.

Process. Let the element *tian* be the number of bundles in a side of the base. From the statement we have 4080 for the negative *shi*, 2 for the positive *fang*, 3 for the positive *lian*, and 1 for the positive *yu*, a cubic expression [2] whose root is the required number.

[Notes]

[1] Li Shanlan, in his *Ze Gu Xi Zhai*, calls the *san jiao luo yi* (dropping one) form a triangular pile of the third degree. The formula is as follows:

$S = \frac{n(n+1)(n+2)}{2 \cdot 3}$. (C) The formula of the volume of the pile of hay with n layers is as follows: $\sum_{i=1}^{n} i = \frac{1}{2} n(n+1)$. A pile in the *luo yi* form is also called a triangular pile. The formula of the volume of the pile in the *luo yi* form with n layers is as follows:

$S = \sum_{i=1}^{n} \frac{1}{2} i(i+1) = \frac{1}{3!} n(n+1)(n+2)$. According to the statement, we have $\frac{1}{3!} n(n+1)(n+2) = 680$. (G)

[2] The expression in modern form is the equation: $x^3 + 3x^2 + 2x - 4080 = 0$. (C)

2.

【原文】

今有茭草一千八百二十束，欲令撒星形埵之。[1] 问：底子几何？

答曰：一十三束。

术曰：立天元一为撒星底子，如积求之。得四万三千六百八十为益实，六为从方，一十一为从上廉，六为从下廉，一为正隅，三乘方开之。[2] 合问。

【注释】

[1]李善兰在其《则古昔斋算学》中称撒星形垛为一个四次的三角垛。它并非一个单一的垛，而是由几个具有三角底子的垛由下向上排列而成的。其求积公式为：$S=\frac{n(n+1)(n+2)(n+3)}{1\cdot 2\cdot 3\cdot 4}$。（陈）撒星形垛又称为三角落一形垛。$n$层撒星形垛的求积公式是：$\sum_{i=1}^{n}\frac{1}{3!}i(i+1)(i+2)=\frac{1}{4!}n(n+1)(n+2)(n+3)$。据题意，$\frac{1}{4!}n(n+1)(n+2)(n+3)=1820$。（郭）

[2] 开方式的现代形式为：$x^4+6x^3+11x^2+6x-43680=0$。（陈）

【今译】

今有1820束茭草，欲垛成撒星形垛。问：其底子为多少束？

答：13束。

术：设天元一为撒星形垛的底子，以如积方法求其解。得到−43680为常数项，6为一次项系数，11为二次项系数，6为三次项系数，1为最高次项系数，开四次方，即得。符合所问。

2. Find a side of the base of a pile consisting of 1820 bundles in the *sa xing* form[1].

Ans. 13 bundles.

Process. Let the element *tian* be a side of the base. From the statement we have 43680 for the negative *shi*, 6 for the positive *fang*, 11 for the positive upper *lian*, 6 for the positive lower *lian*, and 1 for the positive *yu*, an expression [2] of the fourth degree whose root is the required side.

[Notes]

[1] Li Shanlan, in his *Ze Gu Xi Zhai*, calls the *sa xing*(*sa*, loose; *xing*, star)form "triangular piles of the fourth degree." It is not a single pile but several piles with triangular bases ranging from one upward. The formula is as follows:

$S = \frac{n(n+1)(n+2)(n+3)}{1 \cdot 2 \cdot 3 \cdot 4}$.(C)The pile of hay in the *sa xing* form is also called the triangular pile of hay in the *luo yi* form. The formula of the volume of the pile in the *sa xing* form with n layers is $\sum_{i=1}^{n} \frac{1}{3!} i(i+1)(i+2) = \frac{1}{4!} n(n+1)(n+2)(n+3)$. According to the statement, we have $\frac{1}{4!} n(n+1)(n+2)(n+3) = 1820$. (G)

[2] The expression in modern form is the equation: $x^4 + 6x^3 + 11x^2 + 6x - 43680 = 0$. (C)

3.

【原文】

今有茭草三千三百六十七束，欲令岚峰形垛之。[1] 问：底子几何？

答曰：一十二束。

术曰：立天元一为岚峰底子，如积求之。得八万八百八为益实，二为从方，九为从上廉，十为从下廉，三为从隅，三乘方开之。[2] 合问。

【注释】

[1] 李善兰称岚峰形垛为二乘之垛的第一垛。其求积公式为：

$S = \dfrac{n(n+1)(n+2)(3n+1)}{1\cdot 2\cdot 3\cdot 4}$。次数根据 $n(n+1)(n+2)$ 而定，不包括因子 $(3n+1)$。（陈）n 层岚峰形垛的求积公式 $S=\sum_{i=1}^{n}\frac{1}{2!}i(i+1)i=\frac{1}{4!}n(n+1)(n+2)(3n+1)$。据题意，$\frac{1}{4!}n(n+1)(n+2)(3n+1)=3367$。（郭）

[2] 开方式的现代形式为：$3x^4+10x^3+9x^2+2x-80808=0$。（陈）

【今译】

今有 3367 束茭草，欲垛成岚峰形垛。问：其底子为多少束？

答：12 束。

术：设天元一为岚峰形垛的底子，以如积方法求其解。得到 −80808 为常数项，2 为一次项系数，9 为二次项系数，10 为三次项系数，3 为最高次项系数，开四次方，即得。符合所问。

4.

【原文】

今有茭草八千五百六十八束，欲令撒星更落一形垛之。[1] 问：底子几何？

答曰：一十四束。

3. Find a side of the base of a pile of hay of 3367 bundles in the *lan feng* form[1].

Ans. 12 bundles.

Process. Let the element *tian* be a side of the base. From the statement we have 80808 for the negative *shi*, 2 for the positive *fang*, 9 for the positive upper *lian*, 10 for the positive lower *lian*, and 3 for the positive *yu*, a biquadratic expression [2] whose root is the required side.

[Notes]

[1] Li Shanlan calls the *lan feng* (*lan*, mountain-mist; *feng*, summit) form the first pile of the *er cheng zhi duo* (variation of the third degree of a triangular base pile). The formula is $S = \frac{n(n+1)(n+2)(3n+1)}{1 \cdot 2 \cdot 3 \cdot 4}$. The degree is according to the factors $n(n+1)(n+2)$, the factor $(3n+1)$ being not included. (C) The formula of the volume of the pile in the *lan feng* form is as follows: $S = \sum_{i=1}^{n} \frac{1}{2!} i(i+1)i = \frac{1}{4!} n(n+1)(n+2)(3n+1)$. According to the statement, we have $\frac{1}{4!} n(n+1)(n+2)(3n+1) = 3367$. (G)

[2] The expression in modern form is the equation: $3x^4 + 10x^3 + 9x^2 + 2x - 80808 = 0$. (C)

4. Find a side of the base of a pile of hay of 8568 bundles in the *sa xing geng luo yi* form [1].

Ans. 14 bundles.

Process. Let the element *tian* be a side of the base. From the statement we

术曰：立天元一为撒星更落一底子，如积求之。得一百二万八千一百六十为益实，二十四为从方，五十为从上廉，三十五为从二廉，一十为从三廉，一为正隅，四乘方开之。[2] 合问。

【注释】

[1] 撒星更落一形垛又称为三角撒星形垛。n 层更落一形垛的求积公式是 $S=\sum_{i=1}^{n}\frac{1}{4!}i(i+1)(i+2)(i+3)=\frac{1}{5!}n(n+1)(n+2)(n+3)(n+4)$。据题意，$\frac{1}{5!}n(n+1)(n+2)(n+3)(n+4)=8568$。(郭)

[2] 开方式的现代形式为：$x^5+10x^4+35x^3+50x^2+24x-1028160=0$。(陈)

【今译】

今有8568束茭草，欲垛成撒星更落一形垛。问：其底子为多少束？

答：14束。

术：设天元一为撒星更落一形垛的底子，以如积方法求其解。得到 −1028160 为常数项，24为一次项系数，50为二次项系数，35为三次项系数，10为四次项系数，1为最高次项系数，开五次方，即得。符合所问。

5.

【原文】

今有茭草五万三百八十八束，欲令岚峰更落一形捶之。[1] 问：底子几何？

答曰：一十六束。

术曰：立天元一为岚峰更落一底子，如积求之。得六百四万六千五百六十为益实，六为从方，三十五为从上廉，五十为从二廉，二十五为从三廉，四为正隅，四乘方开之。[2] 合问。

have 1028160 for the negative *shi*, 24 for the positive *fang*, 50 for the positive upper *lian*, 35 for the positive second *lian*, 10 for the positive third *lian*, and 1 for the positive *yu*, an expression [2] of the fifth degree whose root is the required side.

[Notes]

[1] The pile of hay in the *sa xing geng luo yi* form is also called triangular pile of hay in the *sa xing* form. The formula of the volume of the pile with n layers is as follows: $S = \sum_{i=1}^{n} \frac{1}{4!} i(i+1)(i+2)(i+3) = \frac{1}{5!} n(n+1)(n+2)(n+3)(n+4)$. According to the statement, $\frac{1}{5!} n(n+1)(n+2)(n+3)(n+4) = 8568$. (G)

[2] The expression in modern form is the equation: $x^5 + 10x^4 + 35x^3 + 50x^2 + 24x - 1028160 = 0$. (C)

5. Find a side of the base of a pile of hay of 50388 bundles in the *lan feng geng luo yi* form [1].

Ans. 16 bundles.

Process. Let the element *tian* be a side of the base. From the statement we have 6046560 for the negative *shi*, 6 for the positive *fang*, 35 for the positive upper *lian*, 50 for the positive second *lian*, 25 for the positive third *lian*, and 4 for the positive *yu*, an expression [2] of the fifth degree whose root is the required side.

【注释】

[1]根据李善兰在其《则古昔斋算学》所讲，岚峰更落一形垛被称为三角底子四次变垛之第一垛。其求积公式为：

$S=\dfrac{n(n+1)(n+2)(n+3)(4n+1)}{1\cdot2\cdot3\cdot4\cdot5}$。次数根据因子 $n(n+1)(n+2)(n+3)$ 而定，并不包括因子 $(4n+1)$。（陈）岚峰更落一形垛又称为三角岚峰形垛。n 层岚峰更落一形垛的求积公式是：$\sum_{i=1}^{n}\frac{1}{3!}i(i+1)(i+2)i=\frac{1}{5!}n(n+1)(n+2)(n+3)(4n+1)$。据题意，$\frac{1}{5!}n(n+1)(n+2)(n+3)(4n+1)=50388$。（郭）

[2]开方式的现代形式为：$4x^5+25x^4+50x^3+35x^2+6x-6046560=0$。（陈）

【今译】

今有50388束茭草，欲垛成岚峰更落一形垛。问：其底子为多少束？

答：16束。

术：设天元一为岚峰更落一形垛的底子，以如积方法求其解。得到-6046560为常数项，6为一次项系数，35为二次项系数，50为三次项系数，25为四次项系数，4为最高次项系数，开五次方，即得。符合所问。

6.

【原文】

今有茭草一垛，直钱二十五贯五百七十八文。只云最上一束直钱九文，次下层层每束累贯三文。[1]问：底子几何？[2]

答曰：二十八束。

术曰：立天元一为茭草底子，如积求之。得一十五万三千四百六十八为益实，二十一为从方，二十七为从廉，六为从隅，立方开之。[3]合问。

【 Notes 】

[1] The *lan feng geng luo yi* (dropping one of the summit) form, according to Li Shanlan in his *Ze Gu Xi Zhai*, is called the first pile of the variation of the fourth degree of a triangular base pile. The formula is $S = \frac{n(n+1)(n+2)(n+3)(4n+1)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}$. The degree is according to the factors $n(n+1)(n+2)(n+3)$, the factor $(4n+1)$ being not included. (C) The pile of hay in the *lan feng geng luo yi* form is also called the triangular pile of hay in the *lan feng* form. The formula of the volume of the pile with n layers is $\sum_{i=1}^{n} \frac{1}{3!} i(i+1)(i+2)i = \frac{1}{5!} n(n+1)(n+2)(n+3)(4n+1)$. According to the statement, $\frac{1}{5!} n(n+1)(n+2)(n+3)(4n+1) = 50388$. (G)

[2] The expression in modern form is the equation: $4x^5 + 25x^4 + 50x^3 + 35x^2 + 6x - 6046560=0$. (C)

6. The whole cost of a pile of hay is 25578 cash. The top bundle costs 9 cash, but each bundle in the layer next below costs 3 additional cash, and so on with each layer. [1] Find the base of the pile. [2]

Ans. 28 bundles.

Process. Let the element *tian* be the number of bundles in the base. From the statement we have 153468 for the negative *shi*, 21 for the positive *fang*, 27 for the positive *lian*, and 6 for the positive *yu*, a cubic expression [3] whose root is the required number.

【注释】

[1]此是茭草垛的各项依次乘以某等差级数各项的求和问题。设等差级数首项 a、公差 d，则其求和公式为 $S=\frac{1}{3!}n(n+1)[2dn+(3a-2d)]$。此问中 $a=9$，$d=3$，则 n 层茭草形垛的钱数是：$\frac{1}{2\times3}n(n+1)(2\times3n+3\times9-2\times3)=25578$。（郭）

[2]此为三角垛，其公式为：$S=\frac{n(n+1)}{1\cdot2}$。（陈）

[3]开方式的现代形式为：$6x^3+27x^2+21x-153468=0$。（陈）

【今译】

今有一垛茭草，值25贯578文。只云最上一束值9文钱，以下层层每束贵3文。问：其底子为多少束？

答：28束。

术：设天元一为茭草垛的底子，以如积方法求其解。得到−153468为常数项，21为一次项系数，27为二次项系数，6为最高次项系数，开立方，即得。符合所问。

7.

【原文】

今有茭草一垛，直钱四十二贯八百四十六文。只云最下每束直钱六文，次上层层每束累贵五文。[1] 问：底子几何？[2]

答曰：三十六束。

术曰：立天元一为茭草底子，如积求之。得二十五万七千七十六为益实，一十三为从方，一十八为从廉，五为从隅，立方开之。[3] 合问。

【Notes】

[1] It is a sum problem to multiply every term of a pile of hay by every term of an arithmetic series in order. Let the first term of the arithmetic series be a, and the difference d. The formula of the sum of the arithmetic series is $S = \frac{1}{3!} n (n+1)[2dn + (3a - 2d)]$. In this problem, $a = 9$, $d = 3$. The cost of the pile of hay with n layers is $\frac{1}{2 \times 3} n (n+1)(2 \times 3n + 3 \times 9 - 2 \times 3) = 25578$. (G)

[2] The pile is in the form of a triangle, the formula being $S = \frac{n(n+1)}{1 \cdot 2}$. (C)

[3] The expression in modern form is the equation: $6x^3 + 27x^2 + 21x - 153468 = 0$. (C)

7. The whole cost of a pile of hay is 42846 cash. Each bundle of the bottom layer is 6 cash, but each bundle of each succeeding layer costs 5 cash more than those in the layer below. [1] Find the base. [2]

Ans. 36 bundles.

Process. Let the element *tian* be the number of bundles in the base. From the statement we have 257076 for the negative *shi*, 13 for the positive *fang*, 18 for the positive *lian*, and 5 for the positive *yu*, a cubic expression [3] whose root is the required base.

【注释】

［1］此亦是茭草垛各项依次乘某等差级数各项的求和问题。此问中该等差级数首项为6＋（$n-1$）×5，公差为−5，故n层茭草形垛的钱数是：$\frac{1}{2\times 3}n(n+1)(5n+3\times 6-5)=42846$。（郭）

［2］此为三角垛，其公式为：$S=\frac{n(n+1)}{1\cdot 2}$。（陈）

［3］开方式的现代形式为：$5x^3+18x^2+13x-257076=0$。（陈）

【今译】

今有一垛茭草，值42贯846文。只云最下层每束值6文钱，以上层层每束贵5文。问：其底子为多少束？

答：36束。

术：设天元一为茭草垛的底子，以如积方法求其解。得到−257076为常数项，13为一次项系数，18为二次项系数，5为最高次项系数，开立方，即得。符合所问。

[Notes]

[1] It is also a sum problem to multiply every term of a pile of hay by every term of an arithmetic series in order. In this problem, the first term of the arithmetic series is $6 + (n - 1) \times 5$, and the difference is -5. Therefore the cost of the pile of hay with n layers is $\frac{1}{2 \times 3} n(n + 1)(5n + 3 \times 6 - 5) = 42846$. (G)

[2] This pile is in the form of a triangle, the formula being $S = \frac{n(n+1)}{1 \cdot 2}$. (C)

[3] The expression in modern form is the equation: $5x^3 + 18x^2 + 13x - 257076 = 0$. (C)

箭积交参　七问

1.

【原文】

今有方、圆箭各一束，共积九十七只。[1] 只云：方箭外周不及圆箭外周四只。[2] 问：方、圆周各几何？

答曰：圆周二十四只，方周二十只。

术曰：立天元一为圆箭外周，如积求之。得四千六百八为益实，二十四为从方，七为从隅，平方开之，[3] 得圆周。合问。

【注释】

[1] 记圆箭数、方箭数分别为 S_1，S_2，此即：$S_1 + S_2 = 97$。（郭）

[2] 记圆箭外周、方箭外周分别为 l_1，l_2，此即：$l_1 - l_2 = 4$。（郭）

[3] 开方式的现代形式为：$7x^2 + 24x - 4608 = 0$。（陈）

【今译】

今有方箭、圆箭各一束，共有97只。只云：方箭的外周比圆箭的外周少4只。问：方箭、圆箭的外周各为多少？

答：圆箭外周24只，方箭外周20只。

术：设天元一为圆箭外周，以如积方法求其解。得到−4608为常数项，24为一次项系数，7为最高次项系数，开平方，得到圆箭外周。符合所问。

2.

【原文】

今有方、圆箭各一束，共积六十二只。[1] 只云：二周相和得三十四只[2]。问：方、圆周各几何？

Jian Ji Jiao Can (Bundles of Arrows)

7 Problems

1. The number of arrows in two bundles, one square and the other round, is 97.[1] The perimeter of the square bundle contains 4 less than the circumference of the round bundle.[2] Find the number of arrows in the perimeter and in the circumference.

Ans. Circumference, 24; perimeter, 20.

Process. Let the element *tian* be the arrows in the circumference of the round bundle. From the statement we have 4608 for the negative *shi*, 24 for the positive *fang*, and 7 for the positive *yu*, a quadratic expression[3] whose root is the required number.

[Notes]

[1] Let the number of arrows in the round bundle be S_1, and the number of arrows in the square bundle S_2. That is, $S_1 + S_2 = 97$. (G)

[2] Let the circumference of arrows in the round bundle be l_1, and the perimeter of arrows in the square bundle l_2. That is, $l_1 - l_2 = 4$. (G)

[3] The expression in modern form is the equation: $7x^2 + 24x - 4608 = 0$. (C)

2. The number of arrows in two bundles, one square and the other round, is 62.[1] The sum of the arrows in the circumferences (the circumference of the round and the perimeter of the square) is 34[2]. Find the number of arrows in the circumferences.

答曰：方周一十六只，圆周一十八只。

术曰：立天元一为方箭外周，如积求之。得二千五百六十为正实，二百七十二为益方，七为正隅，平方开之，[3] 得方周。合问。

【注释】

[1] 记圆箭数、方箭数分别为 S_1，S_2，此即：$S_1 + S_2 = 62$。（郭）

[2] 记圆箭外周、方箭外周分别为 l_1，l_2，此即：$l_1 + l_2 = 34$。（郭）

[3] 开方式的现代形式为：$7x^2 - 272x + 2560 = 0$。（陈）

【今译】

今有方箭、圆箭各一束，共有 62 只。只云：方箭的外周与圆箭的外周之和为 34 只。问：方箭、圆箭的外周各为多少？

答：方箭外周 16 只，圆箭外周 18 只。

术：设天元一为方箭外周，以如积方法求其解。得到 2560 为常数项，−272 为一次项系数，7 为最高次项系数，开平方，得到方箭外周。符合所问。

3.

【原文】

今有方、圆箭各一束，圆箭多如方箭一十二只。[1] 只云：方箭与圆箭外周等。[2] 问：方、圆周各几何？

答曰：周各二十四只。

术曰：立天元一为方、圆箭外周，如积求之。得五百七十六为益实，一为正隅，平方开之。[3] 合问。

Ans. Perimeter, 16; circumference,18.

Process. Let the element *tian* be the arrows in the perimeter of the square bundle. From the statement we have 2560 for the positive *shi*, 272 for the negative *fang*, and 7 for the positive *yu*, a quadratic expression [3] whose root is the required number.

【Notes】

[1] Let the number of arrows in the round bundle be S_1, and the number of arrows in the square bundle S_2. That is, $S_1 + S_2 = 62$. (G)

[2] Let the circumference of arrows in the round bundle be l_1, and the perimeter of arrows in the square bundle l_2. That is, $l_1 + l_2 = 34$. (G)

[3] The expression in modern form is the equation: $7x^2 - 272x + 2560 = 0$. (C)

3. The number of arrows in a round bundle exceeds the number in a square bundle by 12.[1] The number of arrows in the circumference of the round bundle equals the number in the perimeter of the square. [2] Find the arrows in the circumference and in the perimeter.

Ans. Circumference (equal to the perimeter), 24.

Process. Let the element *tian* be the number of arrows in the circumference or the perimeter. From the statement we have 576 for the negative *shi*, and

【注释】

[1] 此即：$S_1 - S_2 = 12$。(郭)

[2] 此即：$l_1 = l_2$。(郭)

[3] 开方式的现代形式为：$x^2 - 576 = 0$。(陈)

【今译】

今有方箭、圆箭各一束，圆箭比方箭多12只。只云：方箭的外周与圆箭的外周相等。问：方箭、圆箭的外周各为多少？

答：方、圆箭外周各24只。

术：设天元一为方、圆箭外周，以如积方法求其解。得到 −576 为常数项，1为最高次项系数，开平方，即得。符合所问。

4.

【原文】

今有方、圆箭各一束，共积九十七只。[1] 只云：方箭外周如圆箭外周六分之五。[2] 问：方、圆周各几何？

答曰：方周二十只，圆周二十四只。

术曰：立天元一为方箭外周，如积求之。得一十一万四千为益实，一千三百二十为从方，二百一十九为从隅，平方开之，[3] 得方周。合问。

【注释】

[1] 此即：$S_1 + S_2 = 97$。(郭)

[2] 此即：$l_2 = \frac{5}{6} l_1$。(郭)

[3] 开方式的现代形式为：$219x^2 + 1320x - 114000 = 0$。(陈)

【今译】

今有方箭、圆箭各一束，共有97只。只云：方箭外周是圆箭外周的 $\frac{5}{6}$。问：方箭、圆箭的外周各为多少？

答：方箭外周20只，圆箭外周24只。

术：设天元一为方箭外周，以如积方法求其解。得到 −114000 为常数项，1320为一次项系数，219为最高次项系数，开平方，得到方箭外周。符合所问。

1 for the positive *yu*, a quadratic expression [3] whose root is the required number.

[Notes]

[1] That is, $S_1 - S_2 = 12$. (G)

[2] That is, $l_1 = l_2$. (G)

[3] The expression in modern form is the equation: $x^2 - 576 = 0$. (C)

4. The number of arrows in two bundles, one round and the other square, is 97.[1] The perimeter contains five-sixths as many arrows as the circumference.[2] Find the number of arrows in the perimeter and in the circumferences.

Ans. Perimeter, 20; circumference, 24.

Process. Let the element *tian* be the number of arrows in the perimeter of the square bundle. From the statement we have 114000 for the negative *shi*, 1320 for the positive *fang*, and 219 for the positive *yu*, a quadratic expression [3] whose root is the required number.

[Notes]

[1] That is, $S_1 + S_2 = 97$. (G)

[2] That is, $l_2 = \frac{5}{6} l_1$. (G)

[3] The expression in modern form is the equation: $219x^2 + 1320x - 114000 = 0$. (C)

5.

【原文】

今有方、圆箭各一束，共积六十二只。[1] 只云：圆箭外周太半与方箭外周强半等。[2] 问：方、圆周各几何？

答曰：圆周一十八只，方周一十六只。

术曰：立天元一为圆箭外周，如积求之。得一万九千四百四十为益实，三百六为从方，四十三为从隅，平方开之，[3] 得圆周。合问。

【注释】

[1] 此即：$S_1 + S_2 = 62$。（郭）

[2] 此即：$\frac{2}{3}l_1 = \frac{3}{4}l_2$。（郭）

[3] 开方式的现代形式为：$43x^2 + 306x - 19440 = 0$。（陈）

【今译】

今有方箭、圆箭各一束，共有62只。只云：圆箭外周的$\frac{2}{3}$与方箭外周的$\frac{3}{4}$相等。问：方箭、圆箭的外周各为多少？

答：圆箭外周18只，方箭外周16只。

术：设天元一为圆箭外周，以如积方法求其解。得到−19440为常数项，306为一次项系数，43为最高次项系数，开平方，得到圆箭外周。符合所问。

6.

【原文】

今有方、圆箭各一束，共积二百八十一只。[1] 只云：圆周四分之一不及方周七分之五八只。[2] 问：方、圆周各几何？

答曰：方周二十八只，圆周四十八只。

术曰：立天元一为方箭外周，如积求之。得四十九万三千一百三十六

5. The number of arrows in two bundles, one round and the other square, is 62.[1] The arrows in the great half of the circumference equal those in the strong half of the perimeter. [2] Find the arrows in the circumference and in the perimeter.

Ans. Circumference, 18; perimeter, 16.

Process. Let the element *tian* be the arrows in the circumference of the round bundle. From the statement we have 19440 for the negative *shi*, 306 for the positive *fang*, and 43 for the positive *yu*, a quadratic expression [3] whose root is the required number.

[Notes]

[1] That is, $S_1 + S_2 = 62$. (G)

[2] That is, $\frac{2}{3}l_1 = \frac{3}{4}l_2$. (G)

[3] The expression in modern form is the equation: $43x^2 + 306x - 19440 = 0$. (C)

6. The number of arrows in two bundles, one round and the other square, is 281.[1] The arrows in one-fourth of the circumference are less than those in five-sevenths of the perimeter by 8. [2] Find the arrows in the circumference and in the perimeter.

Ans. Perimeter, 28; circumference, 48.

为益实，三万一千三百四为益方，一千七百四十七为正隅，平方开之，[3] 得方周。合问。

【注释】

[1] 此即：$S_1 + S_2 = 281$。（郭）

[2] 此即：$\frac{5}{7}l_2 - \frac{1}{4}l_1 = 8$。（郭）

[3] 开方式的现代形式为：$1747x^2 - 31304x - 493136 = 0$。（陈）

【今译】

今有方箭、圆箭各一束，共有281只。只云：圆箭外周的$\frac{1}{4}$比方箭外周的$\frac{5}{7}$少8只。问：方箭、圆箭的外周各为多少？

答：方箭外周28只，圆箭外周48只。

术：设天元一为方箭外周，以如积方法求其解。得到−493136为常数项，−31304为一次项系数，1747为最高次项系数，开平方，得到方箭外周。符合所问。

7.

【原文】

今有方、圆箭各一束，共积二百八只。[1] 只云：圆箭外边第二层周数加二只与方箭外边第一层周数同。[2] 问：方、圆周各几何？

答曰：圆周三十六只，方周三十二只。

术曰：立天元一为圆箭外周，如积求之。得九千九百三十六为益实，二十四为从方，七为从隅，平方开之，[3] 得圆周。合问。

【注释】

[1] 此即：$S_1 + S_2 = 208$。（郭）

Process. Let the element *tian* be the arrows in the perimeter of the square bundle. From the statement we have 493136 for the negative *shi*, 31304 for the negative *fang*, and 1747 for the positive *yu*, a quadratic expression [3] whose root is the required number.

【Notes】

[1] That is, $S_1 + S_2 = 281$. (G)

[2] That is, $\frac{5}{7}l_2 - \frac{1}{4}l_1 = 8$. (G)

[3] The expression in modern form is the equation: $1747x^2 - 31304x - 493136 = 0$. (C)

7. The number of arrows in two bundles, one round and the other square, is 208.[1] The arrows in the circumference of the layer next to the outer layer of the round bundle increased by 2 are equal to those in the perimeter of the square bundle. [2] Find the arrows in the circumference and in the perimeter.

Ans. Circumference, 36; perimeter, 32.

Process. Let the element *tian* be the arrows in the circumference of the round bundle. From the statement we have 9936 for the negative *shi*, 24 for the positive *fang*, and 7 for the positive *yu*, a quadratic expression[3] whose

[2] 此即：$l_2 + 2 = l_1$。（郭）

[3] 开方式的现代形式为：$7x^2 + 24x - 9936 = 0$。（陈）

【今译】

今有方箭、圆箭各一束，共有208只。只云：圆箭外边第二周的只数加2与方箭外周的只数相同。问：方箭、圆箭的外周各为多少？

答：圆箭外周36只，方箭外周32只。

术：设天元一为圆箭外周，以如积方法求其解。得到−9936为常数项，24为一次项系数，7为最高次项系数，开平方，得到圆箭外周。符合所问。

root is the required number.

[Notes]

[1] That is, $S_1 + S_2 = 208$. (G)

[2] That is, $l_2 + 2 = l_1$. (G)

[3] The expression in modern form is the equation: $7x^2 + 24x - 9936 = 0$. (C)

拨换截田 一十九问

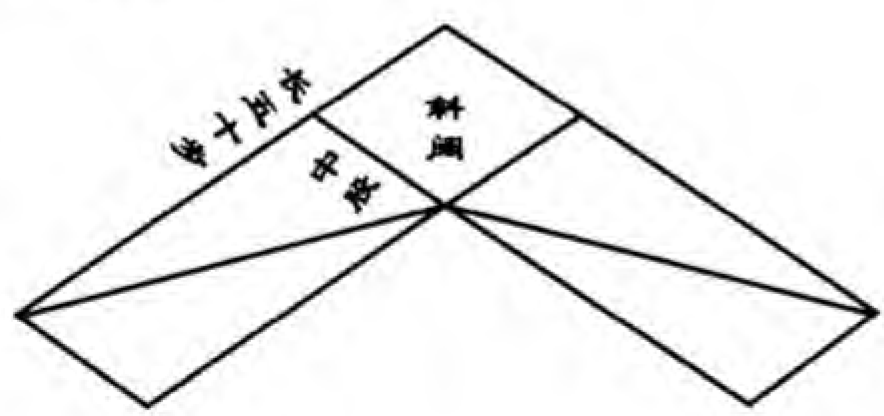
1.

【原文】

今有半种金田一段，长五十步，斜阔一十步。[1] 与邻对拨圭田一段。只云：并圭田长、阔、较，为正实，一十五为益方，一为正隅，平方开之，少如较四步。[2] 问：圭田长、阔各几何？

答曰：长二十五步，阔一十六步。

术曰：立天元一为半种金田之中股，如积求之。得二千四百为正实，一百为从方，五十为益隅，平方开之，[3] 得中股八步。 又：立天元一为较，如积求之。得四千一百七十六为正实，三千三百四十四为益方，六百三十五为从上廉，四十四为益下廉，一为正隅，三乘方开之，[4] 得较。合问。



【注释】

[1] 罗士琳认为此问“仅知斜、长二数，断难立算”“当有脱文”。（郭）

[2] 记圭田的阔、长分别为 a，b，w 为开方式 $w^2-15w+[b+a+(b-a)]=0$ 的根，此即：$(b-a)-w=4$。（郭）

[3] 开方式的现代形式为：$-50x^2+100x+2400=0$。（陈）

[4] 开方式的现代形式为：$x^4-44x^3+635x^2-3344x+4176=0$。（陈）

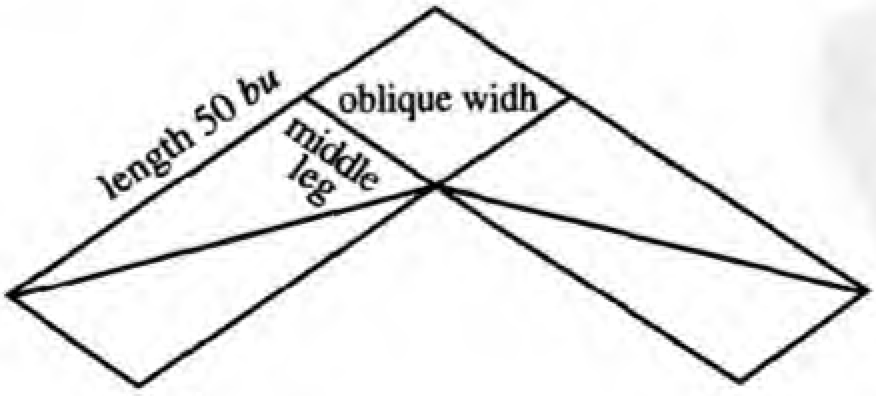
Bo Huan Jie Tian (Land Measurements)

19 Problems

1. The length of half of a *zhong jin tian* is 50 *bu*, and the oblique width 10 *bu*.[1] The owner wishes to trade with his neighbor whose land is in the form of an isosceles triangle. It is said that putting the sum and the difference of the length and the width for the positive *shi*, 15 for the negative *lian*, and 1 for the positive *yu* of a quadratic expression. The root is less than the difference by 4 *bu*.[2] What is the length and the width of the isosceles triangle?

Ans. Length, 25 *bu*; width, 16 *bu*.

Process. Let the element *tian* be the middle leg of half of the *zhong jin tian*. From the statement we have 2400 for the positive *shi*, 100 for the positive *fang*, and 50 for the negative *yu*, a quadratic expression [3] whose root, 8 *bu*, is the middle leg. Again let the element *tian* be the difference. From the statement we have 4176 for the positive *shi*, 3344 for the negative *fang*, 635 for the positive upper *lian*, 44 for the negative lower *lian*, and 1 for the positive *yu*, an expression [4] of the fourth degree whose root is the required difference.



【今译】

今有一块半种金田，长50步，斜阔10步。与邻居对换一块圭田。只云：以圭田的长、阔与长阔差之和作为常数项，−15作为一次项系数，1作为最高次项系数，开平方得到的根比长阔差少4步。问：圭田的长、阔各为多少?

答：圭田的长25步，阔16步。

术：设天元一为半种金田的中股，以如积方法求其解。得到2400为常数项，100为一次项系数，−50为最高次项系数，开平方，得到中股8步。 又：设天元一为圭田的长阔较，以如积方法求其解。得到4176为常数项，−3344为一次项系数，635为二次项系数，−44为益下廉，1为最高次项系数，开四次方，得到长阔较。符合所问。

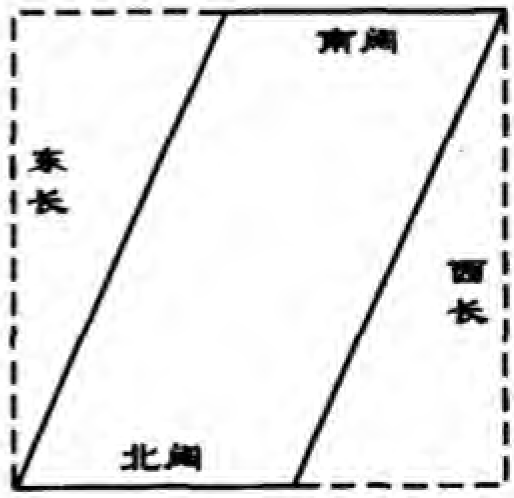
2.

【原文】

今有四不等田一段，东长二十六步，西长二十五步，南阔一十四步，北阔一十七步。[1] 与邻对换直田一段。只云：并直田长、较，为益实，五为从方，一为从隅，平方开之，[2] 所得不及平七步。问：长、平各几何?

答曰：长三十一步，平一十二步。

术曰：立天元一为四不等田之元方面，如积求之。得四百八十为正实，二十八为从方，二为益隅，平方开之，[3] 得二十四步。 又：立天元一为平，如积求之。得七百四十四为益实，一十四为从方，八为益廉，一为正隅，立方开之，[4] 得平。合问。



【Notes】

[1] Luo Shilin concluded that it was difficult to solve the problem when one only knew the oblique and the length. Therefore, he said that some words must be lost in the original text.(G)

[2] Let the width and the length of the isosceles triangle be a and b, and w one of the roots of the equation $w^2 - 15w + [b + a(b - a)] = 0$. That is, $(b - a) - w = 4$. (G)

[3] The expression in modern form is the equation: $-50x^2 + 100x + 2400 = 0$. (C)

[4] The expression in modern form is the equation: $x^4 - 44x^3 + 635x^2 - 3344x + 4176 = 0$. (C)

2. A trapezoid has no two sides equal. The east length is 26 *bu*, the west 25 *bu*, the south width 14 *bu*, and the north 17 *bu*.[1] The owner wishes to trade with his neighbor whose land is in the form of a rectangle. If we take the sum of the length and the difference of the length and the width of the rectangular farm for the negative *shi*, 5 for the positive *fang*, and 1 for the positive *yu*, the root of this quadratic expression is less than the width of the rectangle by 7 *bu*.[2] Find the length and the width of the rectangle.

Ans. Length, 31 *bu*; width, 12 *bu*.

Process. Let the element *tian* be a side of the original square of the trapezoid. From the statement we have 480 for the positive *shi*, 28 for the positive *fang*, and 2 for the negative *yu*, a quadratic expression [3] whose root, 24 *bu*, is a side of the original square. Again let the element *tian* be the width of the rectangle. From the statement we have 744 for the negative *shi*, 14 for the positive *fang*, 8 for the negative *lian*, and 1 for the positive *yu*, a cubic expression [4] whose root is the required width.

【注释】

[1]此问实际上假设四不等田是从一正方形分割出来的。因此，其南、北阔平行，有一对角的顶点恰是正方形的对角的顶点。(郭)

[2]记直田的阔、长分别为a，b，w为开方式$w^2+5w-[b+(b-a)]=0$的根，此即：$a-w=7$。(郭)

[3]开方式的现代形式为：$-2x^2+28x+480=0$。(陈)

[4]开方式的现代形式为：$x^3-8x^2+14x-744=0$。(陈)

【今译】

今有一块四不等田，东长26步，西长25步，南阔14步，北阔17步。与邻居对换一块直田。只云：以直田的长与长阔差相加作为负常数项，5作为一次项系数，1作为最高次项系数，开平方得到的根比直田的阔少7步。问：直田的长、阔各为多少？

答：直田的长31步，阔12步。

术：设天元一为四不等田所在的正方形的边长，以如积方法求其解。得到480为常数项，28为一次项系数，−2为最高次项系数，开平方，得到24步。 又：设天元一为直田的阔，以如积方法求其解。得到−744为常数项，14为一次项系数，−8为二次项系数，1为最高次项系数，开立方，得到阔。符合所问。

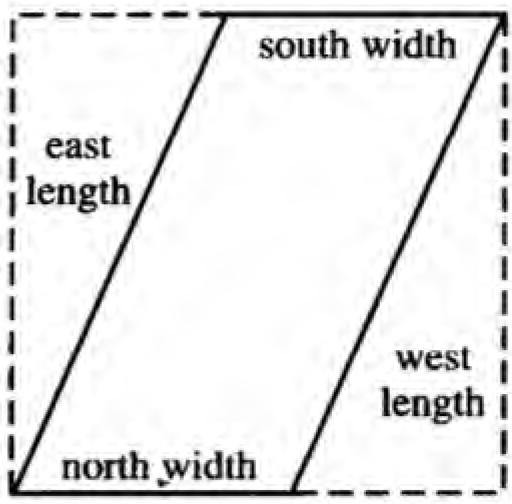
3.

【原文】

今有圭田一段，不云圭阔，只云长五十步，直银五十四两。今从尖截阔一十二步，直银六两。[1] 问：截长及圭阔各几何？

答曰：截长一十六步太半步，阔三十六步。

术曰：立天元一为截长，如积求之。得二千五百为正实，九为益隅，平方开之，[2] 得截长。不尽，按之分术求之。合问。



[Notes]

[1] As a matter of fact, the *si bu deng tian* was formed by cutting from a square land. Therefore, its south width is parallel to the north, and one vertex of a pair of angles on the cross is exactly the vertex of a pair of angles of the square land. (G)

[2]Let the width and the length of the land in the form of a rectangle be a and b, and w one of the roots of the equation $w^2 + 5w - [b + (b - a)] = 0$. That is, $a - w = 7$. (G)

[3] The expression in modern form is the equation: $-2x^2 + 28x + 480 = 0$. (C)

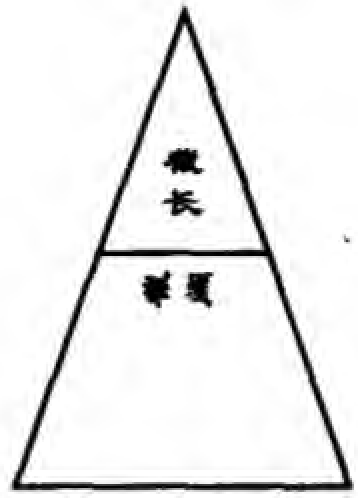
[4] The expression in modern form is the equation: $x^3 - 8x^2 + 14x - 744 = 0$. (C)

3. A farm is in the form of an isosceles triangle with the length equal to 50 *bu*. The price of the farm is 54 taels. A portion whose width is 12 *bu* is cut from the top and sold for 6 taels. [1] What is the length of the portion and what is the width of the given triangle?

Ans. Length of the portion sold, $16\frac{2}{3}$ *bu*;

width of the isosceles triangle, 36 *bu*.

Process. Let the element *tian* be the length of the portion sold. From the



【注释】

［1］圭田是等腰三角形。记圭田的阔、截长分别为 a，b，此即：

$$\frac{54}{\frac{1}{2}\times 50a}=\frac{6}{\frac{1}{2}\times 12b},\ \frac{b}{50}=\frac{12}{a}$$ 。（郭）

［2］开方式的现代形式为：$-9x^2+2500=0$。（陈）

【今译】

今有一块圭田，不知道它的阔，只云长 50 步，值 54 两银。今从尖头阔 12 步处截下，值 6 两银。问：截下部分的长及圭田的阔各为多少？

答：截下的长 $16\frac{2}{3}$ 步，圭田的阔 36 步。

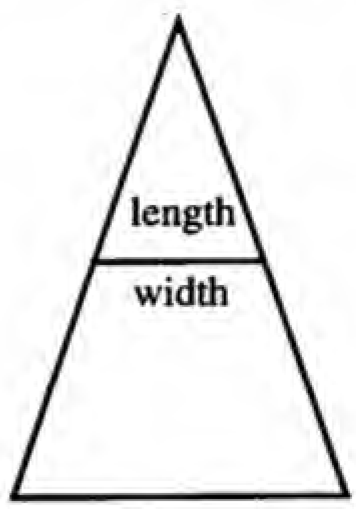
术：设天元一为截下的长，以如积方法求其解。得到 2500 为常数项，−9 为最高次项系数，开平方，得到截长。开方不尽，按照之分法求之。符合所问。

4.

【原文】

今有梯田一段，小阔一十二步，大阔二十步，直钱三十二贯文。今从大头截长四步，直钱九贯五百文。[1] 问：截阔[2] 及元长各几何？

statement we have 2500 for the positive *shi* and 9 for the negative *yu*, a quadratic expression[2] whose root is the required length. Since the root is not integral we shall apply the *zhi fen* method.



[Notes]

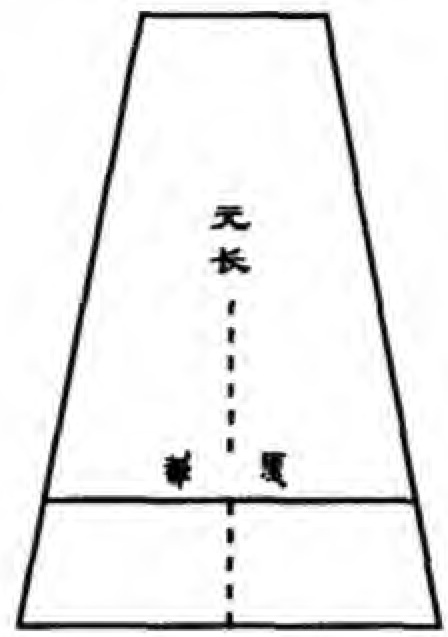
[1] Let the width of the isosceles triangle and the length of the portion sold be a and b. From the statement we have $\dfrac{54}{\frac{1}{2}\times 50a}=\dfrac{6}{\frac{1}{2}\times 12b}$, $\dfrac{b}{50}=\dfrac{12}{a}$. (G)

[2] The expression in modern form is the equation: $-9x^2+2500=0$. (C)

4. A farm in the form of an isosceles trapezoid has its *xiao kuo* equal to 12 *bu* and its *da kuo* equal to 20 *bu*. The price of the farm is 32000 cash. A section cut from the lower base with its length equal to 4 *bu* is worth 9500 cash. [1] What is the *jie kuo*[2] and what is the *yuan chang*?

答曰：截阔一十八步，元长一十六步。

术曰：立天元一为截阔，如积求之。得三百二十四为正实，一为益隅，平方开之，[3] 得截阔。合问。



【注释】

[1] 记梯田的截阔、元长分别为 a_1，b，此即：

$$\frac{32000}{\frac{1}{2}\times(12+20)b}=\frac{9500}{\frac{1}{2}\times(20+a_1)\times 4}，\frac{b-4}{b}=\frac{a_1-12}{20-12}$$ 。（郭）

[2] 截阔为所截部分的上底，元长即为原梯田的高。（陈）

[3] 开方式的现代形式为：$-x^2+324=0$。（陈）

【今译】

今有一块梯田，小阔 12 步，大阔 20 步，值钱 32 贯。今从大头长 4 步处截下，值钱 9 贯 500 文。问：截下部分的阔及梯田原来的长各为多少？

答：截下的阔 18 步，梯田原来的长 16 步。

术：设天元一为截下的阔，以如积方法求其解。得到 324 为常数项，−1 为最高次项系数，开平方，得到截下的阔。符合所问。

Ans. *Jie kuo*, 18 *bu*;

yuan chang, 16 *bu*.

Process. Let the element *tian* be the *jie kuo*. From the statement we have 324 for the positive *shi*, and 1 for the negative *yu*, a quadratic expression [3] whose root is the required *jie kuo*.

【Notes】

[1] Let the *jie kuo* and the *yuan chang* be a_1 and b. From the statement we have

$$\frac{32000}{\frac{1}{2}\times(12+20)b}=\frac{9500}{\frac{1}{2}\times(20+a_1)\times 4},\quad \frac{b-4}{b}=\frac{a_1-12}{20-12}.\ (\mathrm{G})$$

[2] The *jie kuo* (section width) is the upper base of the section. The *yuan chang* is the altitude of the given figure. (C)

[3] The expression in modern form is the equation: $-x^2+324=0$. (C)

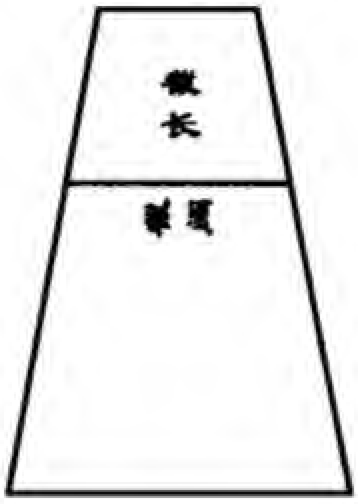
5.

【原文】

今有梯田一段，小阔二十五步，大阔六十五步，正长一百六十步。今从小阔截拨七亩一百一十二步。[1] 问：截长、阔各几何？

答曰：截长五十六步，阔三十九步。

术曰：立天元一为截长，如积求之。得一万四千三百三十六为益实，二百为从方，一为从隅，平方开之，[2] 得截长。合问。



【注释】

[1] 记梯田的截阔、长分别为 a，b，此即：$\frac{1}{2}(25+a)b=1792$，$\frac{b}{160}=\frac{a-25}{65-25}$。（郭）

[2] 开方式的现代形式为：$x^2+200x-14336=0$。（陈）

【今译】

今有一块梯田，小阔 25 步，大阔 65 步，高 160 步。今从小阔起截下 7 亩 112 步，问：截下部分的长、阔各为多少？

答：截下的长 56 步，阔 39 步。

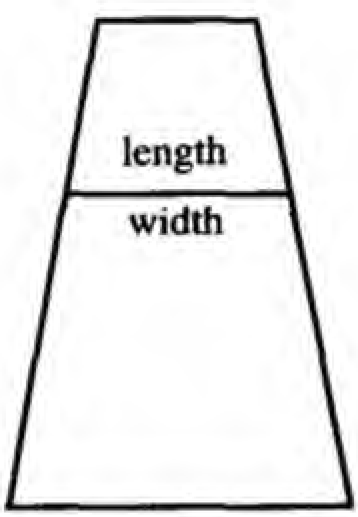
术：设天元一为截下的长，以如积方法求其解。得到 −14336 为常数项，200 为一次项系数，1 为最高次项系数，开平方，得到截下的长。符合所问。

5. A farm in the form of an isosceles trapezoid has its *xiao kuo* equal to 25 *bu*, its *da kuo* 65 *bu*, and its *zheng chang* 160 *bu*. If a section of 7 *mu* 112 *bu* is cut from the upper part what will be the width and the length of the section [1] ?

Ans. Length of the section, 56 *bu*;

width of the section, 39 *bu*.

Process. Let the element *tian* be the length of the section. From the statement we have 14336 for the negative *shi*, 200 for the positive *fang*, and 1 for the positive *yu*, a quadratic expression [2] whose root is the required length.



[Notes]

[1] Let the width and the length of the section cutting from the trapezoid be a and b. From the statement we have $\frac{1}{2}(25+a)b = 1792$, $\frac{b}{160} = \frac{a-25}{65-25}$. (G)

[2] The expression in modern form is the equation: $x^2 + 200x - 14336 = 0$. (C)

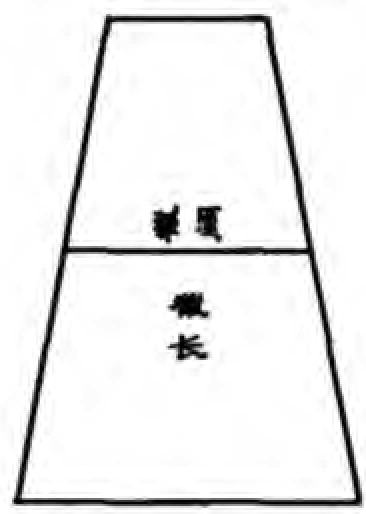
6.

【原文】

今有梯田一段，大阔四十二步，小阔一十八步，正长一百二十步。今从大阔截地十亩一百八十七步二分步之一。[1] 问：截长、阔各几何？

答曰：截长七十五步，截阔二十七步。

术曰：立天元一为截长，如积求之。得二万五千八百七十五为益实，四百二十为从方，一为益隅，平方开之，[2] 得截长。合问。



【注释】

[1]记梯田的截阔、长分别为a, b，此即：$\frac{1}{2}(42+a)b=2587\frac{1}{2}$，$\frac{120-b}{120}=\frac{a-18}{42-18}$。（郭）

[2] 开方式的现代形式为：$-x^2+420x-25875=0$。（陈）

【今译】

今有一块梯田，大阔 42 步，小阔 18 步，高 120 步。今从大阔起截下 10 亩 $187\frac{1}{2}$ 步，问：截下部分的长、阔各为多少？

答：截下的长 75 步，阔 27 步。

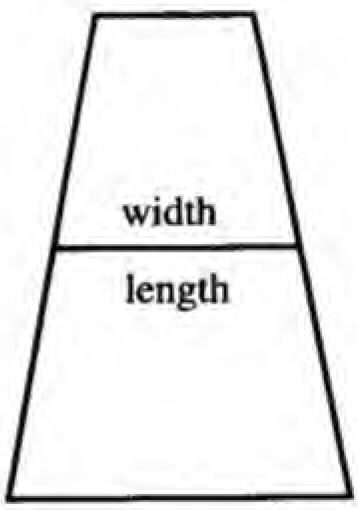
术：设天元一为截下的长，以如积方法求其解。得到 −25875 为常数项，420 为一次项系数，−1 为最高次项系数，开平方，得到截下的长。符合所问。

6. A farm in the form of an isosceles trapezoid has its *da kuo* equal to 42 *bu*, its *xiao kuo* 18 *bu*, and its *zheng chang* 120 *bu*. A section of 10 *mu* $187\frac{1}{2}$ *bu* is cut from the *da kuo*.[1] Find the length and the width of the section.

Ans. Length of the section, 75 *bu*;

width of the section, 27 *bu*.

Process. Let the element *tian* be the length of the section. From the statement we have 25875 for the negative *shi*, 420 for the positive *fang*, and 1 for the negative *yu*, a quadratic expression [2] whose root is the required length.



[Notes]

[1] Let the width and the length of the section cutting from the trapezoid be a and b. From the statement we have $\frac{1}{2}(42 + a)b = 2587\frac{1}{2}$, $\frac{120 - b}{120} = \frac{a - 18}{42 - 18}$. (G)

[2] The expression in modern form is the equation: $-x^2 + 420x - 25875 = 0$. (C)

7.

【原文】

今有圭田一段，长一百三十六步，阔六十八步。今从尖截地二亩四分。[1]问：截长、阔各几何？

答曰：截长四十八步，截阔二十四步。

术曰：立天元一为截长，如积求之。得二千三百四为益实，一为正隅，平方开之，[2]得截长。合问。

【注释】

[1] 记截阔、截长分别为 a, b，此即：$\frac{1}{2}ab = 576$, $\frac{b}{136} = \frac{a}{68}$。（郭）

[2] 开方式的现代形式为：$x^2 - 2304 = 0$。（陈）

【今译】

今有一块圭田，长136步，阔68步。今从尖头起截下$2\frac{2}{5}$亩，问：截下部分的长、阔各为多少？

答：截下的长48步，截下的阔24步。

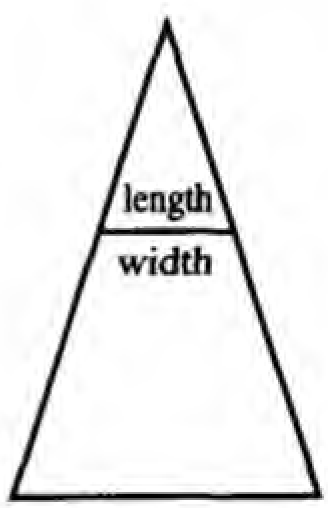
术：设天元一为截下的长，以如积方法求其解。得到−2304为常数项，1为最高次项系数，开平方，得到截下的长。符合所问。

7. From the top of a *gui tian* (a farm is in the form of an isosceles triangle) whose length is 136 *bu* and width 68 *bu* a section of land containing 2 *mu* 4 *fen* is cut off.[1] Find the length and the width of the section.

Ans. Length of the section, 48 *bu*;

width of the section, 24 *bu*.

Process. Let the element *tian* be the length of the section. From the statement we have 2304 for the negative *shi* and 1 for the positive *yu*, a quadratic expression [2] whose root is the required length.



[Notes]

[1] Let the width and the length of the section be a and b. From the statement we have $\frac{1}{2}ab = 576$, $\frac{b}{136} = \frac{a}{68}$.(G)

[2] The expression in modern form is the equation: $x^2 - 2304 = 0$. (C)

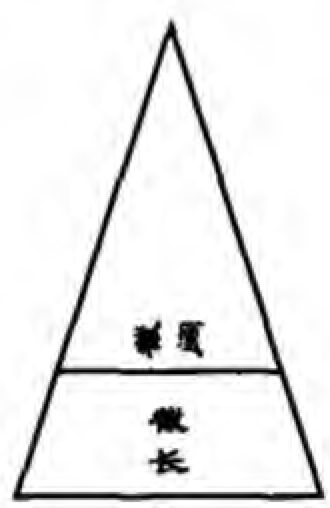
8.

【原文】

今有圭田一段，长一百二十步，阔四十八步。今欲从阔截卖七亩七十五步。[1] 问：截长、阔各几何？

答曰：截长四十五步，截阔三十步。

术曰：立天元一为截长，如积求之。得一千七百五十五为益实，四十八为从方，二分为益隅，平方开之，[2] 得截长。合问。



【注释】

[1] 截下的部分为梯形。记截阔、截长分别为 a、b，此即：$\frac{1}{2}(48+a)b=1755$，$\frac{120-b}{120}=\frac{a}{48}$。（郭）

[2] 开方式的现代形式为：$-\frac{1}{5}x^2+48x-1775=0$。（陈）

【今译】

今有一块圭田，长120步，阔48步。今欲从阔起截下7亩75步，问：截下部分的长、阔各为多少？

答：截下的长45步，截下的阔30步。

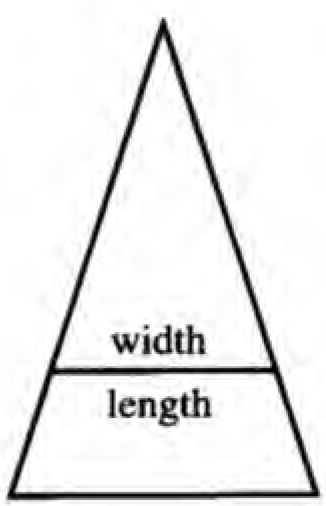
术：设天元一为截下的长，以如积方法求其解。得到 −1755 为常数项，48为从方，$-\frac{1}{5}$ 为最高次项系数，开平方，得到截下的长。符合所问。

8. A section containing 7 *mu* 75 *bu* is cut from the lower part of a *gui tian* whose length is 120 *bu* and width 48 *bu*.[1] Find the length and the width of the section.

Ans. Length of the section, 45 *bu*;

width of the section, 30 *bu*.

Process. Let the element *tian* be the length of the section. From the statement we have 1755 for the negative *shi*, 48 for the positive *fang*, and 2 *fen* for the negative *yu*, a quadratic expression [2] whose root is the required length.



[Notes]

[1] The section is in the form of a trapezoid. Let the width and the length of the section be a and b. From the statement we have

$$\frac{1}{2}(48 + a)b = 1755, \quad \frac{120 - b}{120} = \frac{a}{48}. \text{ (G)}$$

[2] The expression in modern form is the equation: $-\frac{1}{5}x^2 + 48x - 1775 = 0$. (C)

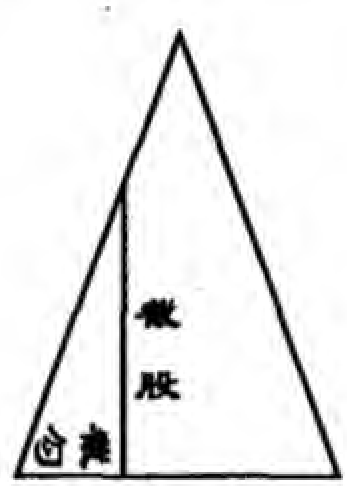
9.

【原文】

今有圭田一段，长一百七十四步，阔一百一十六步。今从东竖截勾股，积三百三十七步半。[1] 问：截勾、股各几何？

答曰：截勾十五步，截股四十五步。

术曰：立天元一为截勾，如积求之。得二百二十五为益实，一为正隅，平方开之，[2] 得截勾。合问。



【注释】

[1] 记截下的勾、股分别为 a，b，此即：$\frac{1}{2}ab=337\frac{1}{2}$，$\frac{b}{174}=\frac{a}{\frac{1}{2}\times 116}$。（郭）

[2] 开方式的现代形式为：$x^2-225=0$。（陈）

【今译】

今有一块圭田，长 174 步，阔 116 步。今从东起截下一勾股形，其面积为 $337\frac{1}{2}$ 步，问：截下的勾、股各为多少？

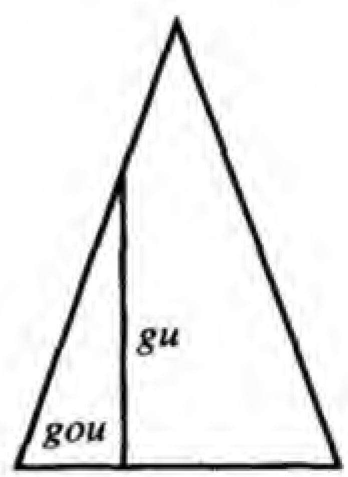
答：截下的勾 15 步，截下的股 45 步。

术：设天元一为截下的勾，以如积方法求其解。得到 -225 为常数项，1 为最高次项系数，开平方，得到截下的勾。符合所问。

9. A section in the form of a right triangle containing $337\frac{1}{2}$ *bu* of land is cut from the east side of a *gui tian* whose altitude is 174 *bu* and base 116 *bu*.[1] Find the *gou* and the *gu* of the section.

Ans. *Gou*, 15 *bu*; *gu*, 45 *bu*.

Process. Let the element *tian* be the *gou*. From the statement we have 225 for the negative *shi* and 1 for the positive *yu*, a quadratic expression [2] whose root is the required *gou*.



[Notes]

[1] Let the *gou* and the *gu* of the section be *a* and *b*. From the statement we have

$$\frac{1}{2}ab = 337\frac{1}{2}, \frac{b}{174} = \frac{a}{\frac{1}{2} \times 116}. \text{ (G)}$$

[2] The expression in modern form is the equation: $x^2 - 225 = 0$. (C)

10.

【原文】

今有勾股田一段，股长八十六步，勾阔二十五步八分。今从尖截卖地一百五十三步六分。[1] 问：截长、阔各几何？

答曰：截长三十二步，阔九步六分。

术曰：立天元一为截长，如积求之。得一千二十四为益实，一为正隅，平方开之，[2] 得截长。合问。

【注释】

[1] "步" 下八分、六分即分别为0.8步、0.6步。记截下的勾、股分别为a，b，此即：$\frac{1}{2}ab = 153.6$，$\frac{b}{86} = \frac{a}{25.8}$。（郭）

[2] 开方式的现代形式为：$x^2 - 1024 = 0$。（陈）

【今译】

今有一块勾股田，股长86步，阔$25\frac{4}{5}$步。今从尖头截下卖掉一勾股田，其面积为$153\frac{3}{5}$步，问：截下股长、勾阔各为多少？

答：截下的股长32步，勾阔9.6步。

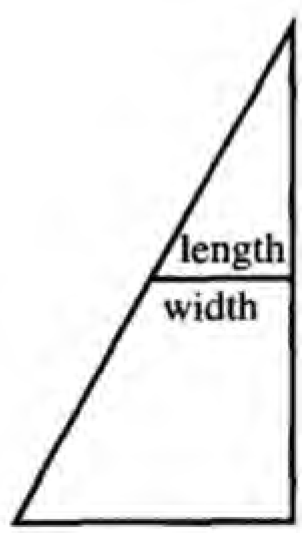
术：设天元一为截下的股长，以如积方法求其解。得到-1024为常数项，1为最高次项系数，开平方，得到截下的股长。符合所问。

10. A farm is in the form of a right triangle whose *gu* is equal to 86 *bu* and *gou* 25 *bu* 8 *fen*. A section containing 153 *bu* 6 *fen* is cut from the top and sold.[1] What are the length and the width of the section?

Ans. Length of the section, 32 *bu*;

width of the section, 9 *bu* 6 *fen*.

Process. Let the element *tian* be the length. From the statement we have 1024 for the negative *shi* and 1 for the positive *yu*, a quadratic expression[2] whose root is the required length.



[Notes]

[1] The 8 *fen* and 6 *fen* following *bu* is 0.8 *bu* and 0.6 *bu* respectively. Let the *gou* and the *gu* of the section be *a* and *b*. From the statement we have $\frac{1}{2}ab = 153.6$, $\frac{b}{86} = \frac{a}{25.8}$. (G)

[2] The expression in modern form is the equation: $x^2 - 1024 = 0$. (C)

11.

【原文】

今有勾股田一段，勾阔五十七步，股长九十五步。今从勾横截地八亩三十七步半。[1] 问：截长、阔各几何？

答曰：截长四十五步，截阔三十步。

术曰：立天元一为截长，如积求之。得六千五百二十五为正实，一百九十为益方，一为正隅，平方开之，[2] 得截长。合问。

【注释】

[1] 截下的为一梯形。记截下的小阔、高分别为 a，b，此即：$\frac{1}{2}(57+a)b=1957\frac{1}{2}$，$\frac{95-b}{95}=\frac{a}{57}$。（郭）

[2] 开方式的现代形式为：$x^2-190x+6525=0$。（陈）

【今译】

今有一块勾股田，勾阔 57 步，股长 95 步。今从勾横截一块地，其面积为 8 亩 $37\frac{1}{2}$ 步，问：截下的长、阔各为多少？

答：截下的长 45 步，阔 30 步。

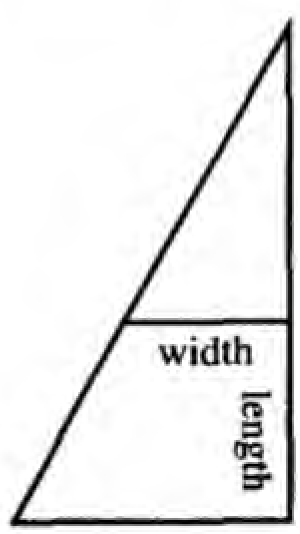
术：设天元一为截下的长，以如积方法求其解。得到 6525 为常数项，−190 为一次项系数，1 为最高次项系数，开平方，得到截下的长。符合所问。

11. A farm is in the form of a right triangle whose *gou* is 57 *bu* and *gu* 95 *bu*. A section containing 8 *mu* $37\frac{1}{2}$ *bu* is cut from the base.[1] What are the length and the width of the section?

Ans. Length of the section, 45 *bu*;

width of the section, 30 *bu*.

Process. Let the element *tian* be the length. From the statement we have 6525 for the positive *shi*, 190 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression [2] whose root is the required length.



[Notes]

[1] The section is in the form of a trapezoid. Let the small width and the altitude of the section be a and b. From the statement we have

$$\frac{1}{2}(57 + a)b = 1957\frac{1}{2},\ \frac{95 - b}{95} = \frac{a}{57}.\ \text{(G)}$$

[2] The expression in modern form is the equation: $x^2 - 190x + 6525 = 0$. (C)

12.

【原文】

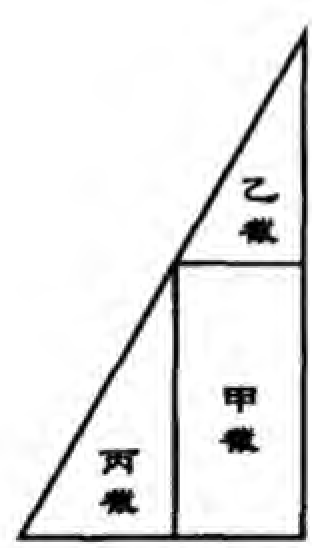
今有勾股田一段，勾阔六十步，股长一百五十步，令甲、乙、丙三人分之：甲截积二千九十步，乙截积一千八百五步，丙截积六百五步。从南横截一勾股与乙，从东竖截一勾股与丙，外剩直田一段与甲。[1] 问：三人各截长、阔各几何？

答曰：甲截长五十五步，截阔三十八步；

乙截股九十五步，截勾三十八步；

丙截股五十五步，截勾二十二步。

术曰：立天元一为乙截勾，如积求之。得一千四百四十四为益实，一为正隅，平方开之，[2] 得乙截勾。即甲截阔。　又：立天元一为丙截股，如积求之。得六百五为益实，二分为从隅，平方开之，[3] 得丙截股。即甲截长。合问。



【注释】

[1] 记乙所截得的小勾、股为 a_1，b_1，丙所截得的小勾、股为 a_2，b_2，此即：$a_1+a_2=60$，$b_1+b_2=150$，$a_1b_2=2090$，$\frac{1}{2}a_1b_1=1805$，$\frac{1}{2}a_2b_2=605$。（郭）

[2] 开方式的现代形式为：$x^2-1444=0$。（陈）

[3] "二分" 即 0.2 步。（郭）开方式的现代形式为：$0.2x^2-605=0$。（陈）

12. A farm, in the form of a right triangle whose *gou* is 60 *bu* and *gu* 150 *bu*, is divided among Jia, Yi, and Bing. Jia receives 2090 *bu*, Yi 1805 *bu*, and Bing 605 *bu*. Yi's section is a right triangle at the south, Bing's section a right triangle at the east, and Jia's, the remaining part, a rectangle. [1] Find the length and the width of each section.

Ans. Jia's section.

length, 55 *bu*;

width, 38 *bu*.

Yi's section.

length, 95 *bu*;

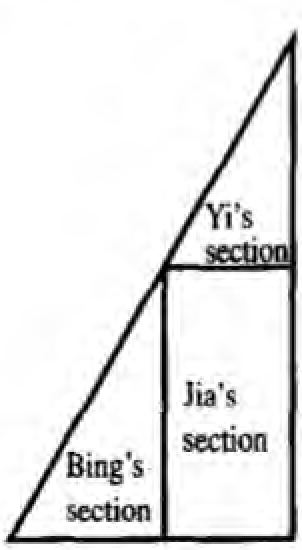
width, 38 *bu*.

Bing's section.

length, 55 *bu*;

width, 22 *bu*.

Process. Let the element *tian* be the *gou* of Yi's section. From the statement we have 1444 for the negative *shi* and 1 for the positive *yu*, a quadratic expression[2] whose root is the *gou* of Yi's section which is also the width of Jia's section. Again let the element tian be the *gu* of Bing's section. From the statement we have 605 for the negative *shi* and 2 *fen* for the positive *yu*, a quadratic expression [3] whose root is the *gu* of Bing's section which is also the length of Jia's section.



【今译】

今有一块勾股田，勾阔60步，股长150步。使甲、乙、丙三人分之：甲截得的面积为2090步，乙截得的面积为1805步，丙截得的面积为605步。从南横截得一勾股形给乙，从东竖截得一勾股形给丙，另外剩下的一块直田给甲。问：三人截下的长、阔各为多少？

答：甲截下的长55步，阔38步；

乙截得的股95步，截得的勾38步；

丙截得的股55步，截得的勾22步。

术：设天元一为乙截得的勾，以如积方法求其解。得到 -1444 为常数项，1为最高次项系数，开平方，得到乙截得的勾。此即甲截得的阔。　又：设天元一为丙截得的股，以如积方法求其解。得到 -605 为常数项，$\frac{1}{5}$ 为最高次项系数，开平方，得到丙截得的股。此即甲截得的长。符合所问。

13.

【原文】

今有梯田一段，正长二百一十步，小阔五十步，大阔九十二步，令甲、乙、丙、丁分之：甲截积六千三百五十二步二分步之一，乙截积五千三十七步二分步之一，丙截积二千一百六十二步二分步之一，丁截积一千三百五十七步二分步之一。从上先截给甲，次与乙、丙、丁。[1] 问：各截长、阔几何？

答曰：甲截长一百五步，截阔七十一步；

乙截长六十五步，截阔八十四步；

丙截长二十五步，截阔八十九步；

丁截长一十五步，截阔九十二步。

术曰：立天元一为甲截长，如积求之。得六万三千五百二十五为益实，

[Notes]

[1] The small *gou* and *gu* of Yi' s section be a_1 and b_1, the small *gou* and *gu* of Bing' s section be a_2 and b_2. From the statement we have $a_1 + a_2 = 60$, $b_1 + b_2 = 150$, $a_1b_2 = 2090$, $\frac{1}{2}a_1b_1 = 1805$, $\frac{1}{2}a_2b_2 = 605$. (G)

[2] The expression in modern form is the equation: $x^2 - 1444 = 0$. (C)

[3] *Fen* is 0.2 *bu*. (G) The expression in modern form is the equation: $\frac{1}{5}x^2 - 605 = 0$. (C)

13. A farm in the form of an isosceles trapezoid has its *zheng chang* 210 *bu*, *xiao kuo* 50 *bu*, and *da kuo* 92 *bu*. The farm is divided among Jia, Yi, Bing, and Ding. Jia receives $6352\frac{1}{2}$ *bu*, Yi $5037\frac{1}{2}$ *bu*, Bing $2162\frac{1}{2}$ *bu*, and Ding $1357\frac{1}{2}$ *bu*. Jia' s section is first received from the top of the farm. Yi's section follows Jia's, Bing's section follows Yi's, and Ding's section follows Bing's. [1] Find the length and the width of each section.

Ans. Jia's section:

length, 105 *bu*;

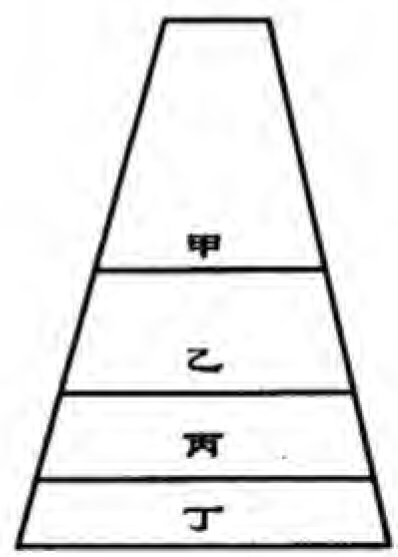
width, 71 *bu*.

Yi's section:

length, 65 *bu*;

width, 84 *bu*.

五百为从方，一为从隅，平方开之，[2] 得甲截长。又：立天元一为乙截长，如积求之。得一万七十五为益实，一百四十二为从方，二分为从隅，平方开之，[3] 得乙截长。又：立天元一为丙截长，如积求之。得四千三百二十五为益实，一百六十八为从方，二分为从隅，平方开之，[4] 得丙截长。又：立天元一为丁截长，如积求之。得二千七百一十五为益实，一百七十八为从方，二分为从隅，平方开之，[5] 得丁截长。合问。



【注释】

[1] 甲、乙、丙、丁所截均为梯形，记其截阔、长分别为 a_1，b_1；a_2，b_2；a_3，b_3；a_4，b_4，此即：$a_4 = 92$，$b_1 + b_2 + b_3 + b_4 = 210$，$\frac{1}{2}(50 + a_1)b_1 = 6352\frac{1}{2}$，$\frac{1}{2}(a_1 + a_2)b_2 = 5037\frac{1}{2}$，$\frac{1}{2}(a_2 + a_3)b_3 = 2162\frac{1}{2}$，$\frac{1}{2}(a_3 + 92)b_4 = 1357\frac{1}{2}$。

$\frac{b_1}{210} = \frac{a_1 - 50}{92 - 50}$，$\frac{b_1 + b_2}{210} = \frac{a_2 - 50}{92 - 50}$，$\frac{b_1 + b_2 + b_3}{210} = \frac{a_3 - 50}{92 - 50}$。（郭）

[2] 开方式的现代形式为：$x^2 + 500 - 63525 = 0$。（陈）

[3] 开方式的现代形式为：$\frac{1}{5}x^2 + 142x - 10075 = 0$。（陈）

[4] 开方式的现代形式为：$\frac{1}{5}x^2 + 168x - 4325 = 0$。（陈）

[5] 开方式的现代形式为：$\frac{1}{5}x^2 + 178x - 2715 = 0$。（陈）

Bing's section:

length, 25 *bu*;

width, 89 *bu*.

Ding's section:

length, 15 *bu*;

width, 92 *bu*.

Process. Let the element *tian* be the length of Jia's section. From the statement we have 63525 for the negative *shi*, 500 for the positive *fang*, and 1 for the positive *yu*, a quadratic expression[2] whose root is the length of Jia's section. Again let the element *tian* be the length of Yi's section. From the statement we have 10075 for the negative *shi*, 142 for the positive *fang*, and 2 *fen* for the positive *yu*, a quadratic expression[3] whose root is the length of Yi's section. Again let the element *tian* be the length of Bing's section. From the statement we have 4325 for the negative *shi*, 168 for the positive *fang*, and 2 *fen* for the positive *yu*, a quadratic expression[4] whose root is the length of Bing's section. Again let the element *tian* be the length of Ding's section. From the statement we have 2715 for the negative *shi*, 178 for the positive *fang*, and 2 *fen* for the positive *yu*, a quadratic expression[5] whose root is the length of Ding's section.

【今译】

今有一块梯田，高210步，小阔50步，大阔92步。使甲、乙、丙、丁分之：甲截得的面积为$6352\frac{1}{2}$步，乙截得的面积为$5037\frac{1}{2}$步，丙截得的面积为$2162\frac{1}{2}$步，丁截得的面积为$1357\frac{1}{2}$步。自上而下先截给甲，再依次给乙、丙、丁。问：截下的长、阔各为多少？

答：甲截下的长105步，截得的阔71步；

乙截得的长65步，截得的阔84步；

丙截得的长25步，截得的阔89步；

丁截得的长15步，截得的阔92步。

术：设天元一为甲截得的长，以如积方法求其解。得到−63525为常数项，500为一次项系数，1为最高次项系数，开平方，得到甲截得的长。 又：设天元一为乙截得的长，以如积方法求其解。得到−10075为常数项，142为一次项系数，$\frac{1}{5}$为最高次项系数，开平方，得到乙截得的长。 又：设天元一为丙截得的长，以如积方法求其解。得到−4325为常数项，168为一次项系数，$\frac{1}{5}$为最高次项系数，开平方，得到丙截得的长。 又：设天元一为丁截得的长，以如积方法求其解。得到−2715为常数项，178为一次项系数，$\frac{1}{5}$为最高次项系数，开平方，得到丁截得的长。符合所问。

14.

【原文】

今有弧田一段，弦长七十步，矢阔二十五步。今从弧背复截弧矢，积二十六步。[1] 问：截弦、矢各几何？

答曰：截弦二十四步，截矢二步。

术曰：先求得圆径七十四步[2]。立天元一为截矢，如积求之。得二千

[Notes]

[1] The sections of Jia, Yi, Bing, and Ding are all in the forms of a trapezoid. Let the width and the length of their sections be a_1 and b_1, a_2 and b_2, a_3 and b_3, a_4 and b_4, respectively. From the statement we have $a_4 = 92$, $b_1 + b_2 + b_3 + b_4 = 210$, $\frac{1}{2}(50 + a_1) b_1 = 6352\frac{1}{2}$, $\frac{1}{2}(a_1 + a_2) b_2 = 5037\frac{1}{2}$, $\frac{1}{2}(a_2 + a_3) b_3 = 2162\frac{1}{2}$, $\frac{1}{2}(a_3 + 92) b_4 = 1357\frac{1}{2}$.

$$\frac{b_1}{210} = \frac{a_1 - 50}{92 - 50}, \quad \frac{b_1 + b_2}{210} = \frac{a_2 - 50}{92 - 50}, \quad \frac{b_1 + b_2 + b_3}{210} = \frac{a_3 - 50}{92 - 50}. \quad (G)$$

[2] The expression in modern form is the equation: $x^2 + 500 - 63525 = 0$. (C)

[3] The expression in modern form is the equation: $\frac{1}{5}x^2 + 142x - 10075 = 0$. (C)

[4] The expression in modern form is the equation: $\frac{1}{5}x^2 + 168x - 4325 = 0$. (C)

[5] The expression in modern form is the equation: $\frac{1}{5}x^2 + 178x - 2715 = 0$. (C)

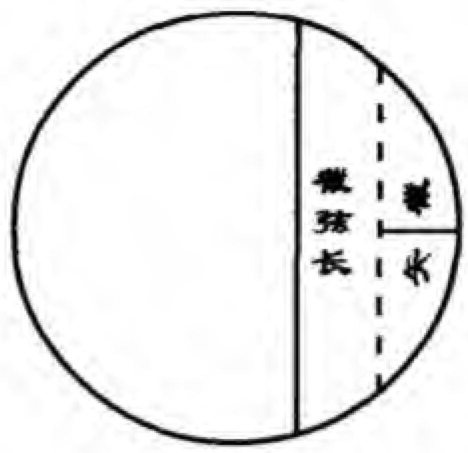
14. From a *hu tian* with its *xian* 70 *bu*, and its *shi* 25 *bu* a smaller *hu shi ji* which equals 26 *bu* is cut.[1] Find the *xian* and *shi* of the *hu shi ji*.

Ans. *Xian*, 24 *bu*;

shi, 2 *bu*.

Process. First find the diameter of the circle which is 74 *bu* [2], then let the element *tian* be the *shi* of the *hu shi ji*. From the statement we have 2704 for the negative *shi*, 104 for the positive upper *lian*, 296 for the positive

七百四为益实，一百四为从上廉，二百九十六为从下廉，五为益隅，三乘方开之，[3] 得截矢二步。自之，以减倍积，余，以矢除之，即弦。[4] 合问。



【注释】

［1］记弧田的面积、弦、矢分别为 S，c，v，根据《九章算术》弧田术，其面积为 $S=\frac{1}{2}(cv+v^2)=1187\frac{1}{2}$。记截弧田的面积、弦、矢分别为 S_1，c_1，v_1，据题意，其面积为 $S_1=\frac{1}{2}(c_1v_1+v_1^2)=26$。（郭）

［2］据刘徽《九章算术注》，依《九章算术》勾股章“勾股锯圆材”之术求弧田所在圆之径 d：$d=\frac{(\frac{c}{2})^2}{v}+v=\frac{(\frac{70}{2})^2}{25}+25=74$。（郭）

［3］开方式的现代形式为：$-5x^4+296x^3+104x^2-2704=0$。（陈）

［4］此即：$c_1=\frac{2S_1-v_1^2}{v_1}=24$。（郭）

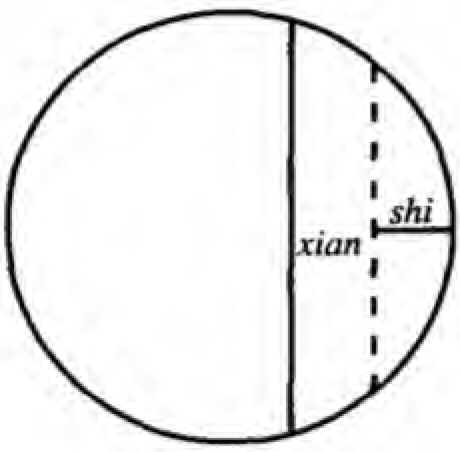
【今译】

今有一段弧田，弦长 70 步，矢阔 25 步。今从弧背再截出一小弧田，其面积为 26 步。问：所截弧田的弦、矢各为多少？

答：截弦 24 步，截矢 2 步。

术：先求得弧田所在圆的直径 74 步。设天元一为截矢，以如积方法求其解。得到 −2704 为常数项，104 为二次项系数，296 为三次项系数，−5 为最高次项系数，开四次方，得到截矢 2 步。截矢自乘，以它减截弧田面积的 2 倍，其余数，以矢除之，便是截弦。符合所问。

lower *lian*, and 5 for the negative *yu*, an expression[3] of the fourth degree whose root, 2 *bu*, is the required *shi*. From twice the given area subtract the square of the *shi*. Divide this remainder by the *shi*. The result is the required *xian* [4].



【Notes】

[1] Let the area, the *xian*, and *shi* of the *hu tian* be S, c, and v. According to the *hu tian* method in *The Nine Chapters of Mathematical Procedures*, the area is $S = \frac{1}{2}(cv + v^2) = 1187\frac{1}{2}$. Let the area, the *xian*, and *shi* of the *hu shi ji* be S_1, c_1, and v_1. From the statement its area is $S_1 = \frac{1}{2}(c_1v_1 + v_1^2) = 26$. (G)

[2] According to Liu Hui's *Commentary of "The Nine Chapters of Mathematical Procedures"* and the *gou gu ju yuan cai* method of in the chapter *gou gu* of *The Nine Chapters of Mathematical Procedures*, we have the diameter of the circle d, that is,

$$d = \frac{(\frac{c}{2})^2}{v} + v = \frac{(\frac{70}{2})^2}{25} + 25 = 74. \text{ (G)}$$

[3] The expression in modern form is the equation: $-5x^4 + 296x^3 + 104x^2 - 2704 = 0$. (C)

[4] That is, $c_1 = \frac{2S_1 - v_1^2}{v_1} = 24$. (G)

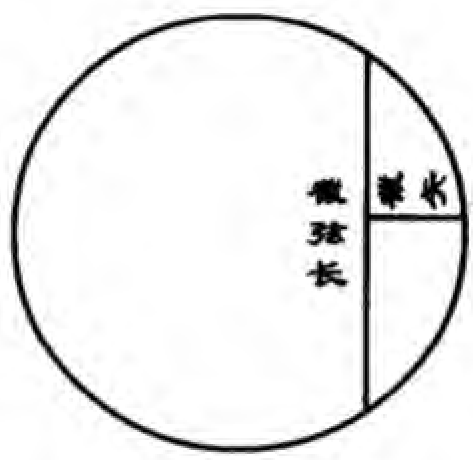
15.

【原文】

今有圆田一段，周二百六十七步。今从边截一弧，计积一千三百一十二步中半步。[1] 问：截弦、矢各几何？

答曰：截矢二十五步，截弦八十步。

术曰：立天元一为截矢，如积求之。得六百八十九万六百二十五为正实，五千二百五十为益上廉，三百五十六为益下廉，五为正隅，三乘方开之，[2] 得截矢。合问。



【注释】

[1] 此即：$S=\frac{1}{2}(cv+v^2)=1312\frac{1}{2}$。（郭）

[2] 开方式的现代形式为：$5x^4-356x^3-5250x^2+6890625=0$。（陈）

【今译】

今有一段圆田，周长267步。今从边截出一弧田，计其面积为$1312\frac{1}{2}$步。问：所截弧田的弦、矢各为多少？

答：截矢25步，截弦80步。

术：设天元一为截矢，以如积方法求其解。得到6890625为常数项，-5250为二次项系数，-356为三次项系数，5为最高次项系数，开四次方，得到截矢。符合所问。

15. The circumference of a circular piece of land is 267 *bu*. A segment is cut off with an area equal to $1312\frac{1}{2}$ *bu*. [1] Find the *xian* and the *shi*.

Ans. *Shi* of the segment, 25 *bu*;

xian, 80 *bu*.

Process. Let the element *tian* be the *shi* of the segment. From the statement we have 6890625 for the positive *shi*, 5250 for the negative upper *lian*, 356 for the negative lower *lian*, and 5 for the positive *yu*, an expression [2] of the fourth degree whose root is the required *shi*.

【Notes】

[1] That is, $S = \frac{1}{2}(cv + v^2) = 1312\frac{1}{2}$. (G)

[2] The expression in modern form is the equation: $5x^4 - 356x^3 - 5250x^2 + 6890625 = 0$. (C)

16.

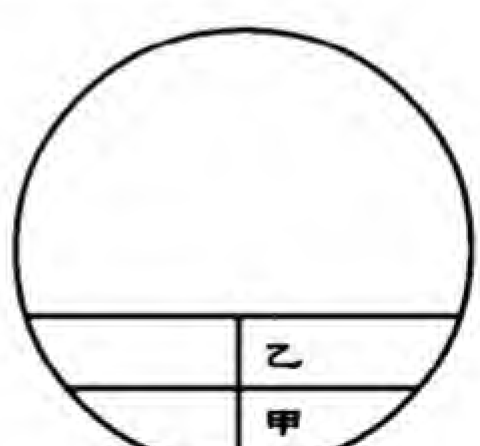
【原文】

今有圆田一段，径九十步。甲、乙共截一弧。其甲从边复截一弧，以次给乙。甲截积二百八十三步二分步之一，乙截积五百二十六步二分步之一。[1] 问：甲、乙各截弦、矢几何？

答曰：甲截矢九步，截弦五十四步；

乙截矢九步，截弦七十二步。

术曰：立天元一为甲截矢，如积求之。得三十二万一千四百八十九为正实，一千一百三十四为益上廉，三百六十为益下廉，五为正隅，三乘方开之，[2] 得甲截矢九步。列甲积，倍之，通分内子，内减矢幂，余以矢除之，即甲截弦。[3] 又：立天元一为共截矢，如积求之。得二百六十二万四千四百为益实，三千二百四十为从上廉，三百六十为从二廉，五为益隅，三乘方开之，[4] 得共截矢一十八步。内减甲截矢，余即乙截矢。 又：共矢自之，以减甲、乙并积倍之通分内子之数，余，以共矢而一，即乙截弦。[5] 合问。



【注释】

[1] 记甲所截及甲乙共截之弧田的弦、矢分别为 c_1, v_1; c_2, v_2，甲、乙所截面积分别为 S_1, S_2，此即：$S_1 = \frac{1}{2}(c_1v_1 + v_1^2) = 283\frac{1}{2}$，

16. A tract of land is in the form of a circle whose diameter is 90 *bu*. Jia and Yi each own a segment. Jia's share is the segment of one base and contains $283\frac{1}{2}$ *bu*. Yi's share is the segment of two bases and contains $526\frac{1}{2}$ *bu*.[1] Find the *xian* and the *shi* of each segment.

Ans. Jia's *shi*, 9 *bu*;

xian, 54 *bu*.

Yi's *shi*, 9 *bu*;

xian, 72 *bu*.

Process. Let the element *tian* be the *shi* of Jia's segment. From the statement we have 321489 for the positive *shi*, 1134 for the negative upper *lian*, 360 for the negative lower *lian*, and 5 for the positive *yu*, an expression[2] of the fourth degree whose root, 9 *bu*, is the required *shi* of Jia's segment. Reduce the area of Jia's segment to an improper fraction, double it, and from the numerator subtract the square of the *shi*. Divide this remainder by the *shi*. The result is the required *xian*.[3] Again let the element tian be the *shi* of Jia's and Yi's segments taken together. From the statement we have 2624400 for the negative *shi*, 3240 for the positive upper *lian*, 360 for the positive second *lian*, and 5 for the negative *yu*, an expression[4] of the fourth degree whose root, 18 *bu*, is the required *shi*. Subtract from this root the *shi* of Jia's segment; the result is the *shi* of Yi's segment. Reduce to improper fractions the double Jia's and Yi's segments and from the sum of the numerators subtract the square of the large *shi* (Jia's *shi* plus Yi's *shi*). Dividing this result by the large *shi* we have the *xian* of Yi's segment[5].

$S_2=\frac{1}{2}(c_2v_2+v^2_2)-\frac{1}{2}(c_1v_1+v^2_1)=526\frac{1}{2}$。(郭)

[2] 开方式的现代形式为：$5x^4-360x^3-1134x^2+321489=0$。(陈)

[3] 这里指从甲截积的2倍中减去矢幂，然后以矢除余数。(陈) 原文脱“倍之”二字，依意补。(郭)

[4] 开方式的现代形式为：$-5x^4+360x^3+3240x^2-2624400=0$。(陈)

[5] 原文脱“倍之”二字。依意补。此即：$c_2=\frac{2(S_1+S_2)-v_2^2}{v_2}=72$。(郭)

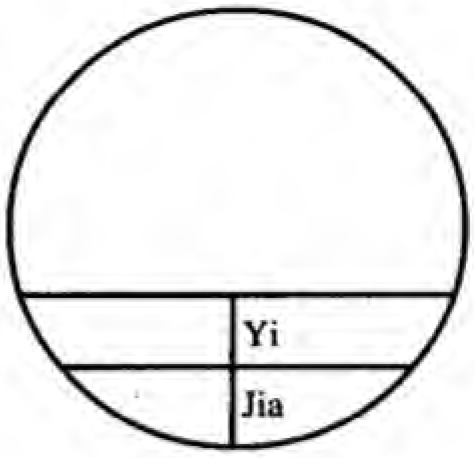
【今译】

今有一段圆田，直径90步。甲、乙共同从圆上截一弧田。甲从边上先截一小弧田，剩下的给乙。甲截得的面积为$283\frac{1}{2}$步，乙截得的面积为$526\frac{1}{2}$步。问：甲、乙各截的弦、矢为多少？

答：甲截矢9步，截弦54步；

乙截矢9步，截弦72步。

术：设天元一为甲截矢，以如积方法求其解。得到321489为常数项，−1134为二次项系数，−360为三次项系数，5为最高次项系数，开四次方，得到甲截矢9步。列出甲所截的面积，加倍，通分内子，内中减去矢的平方，其余数，以矢除之，就是甲截弦。又：设天元一为共截矢，以如积方法求其解。得到−2624400为常数项，3240为二次项系数，360为三次项系数，−5为最高次项系数，开四次方，得到共截矢18步。内中减去甲截矢，就是乙截矢。又：甲乙共矢自乘，以其减甲乙共截得的面积2倍的通分内子后所得的数，其余数，以甲乙共矢除之，就是乙截弦。符合所问。



[Notes]

[1] Let the *xian* and the *shi* of Jia's segment be c_1 and v_1, the *xian* and the *shi* of the segment that Jia and Yi shared c_2 and v_2, and the areas of Jia and Yi's segments S_1 and S_2. From the statement we have $S_1 = \frac{1}{2}(c_1v_1 + v_1^2) = 283\frac{1}{2}$,

$$S_2 = \frac{1}{2}(c_2v_2 + v_2^2) - \frac{1}{2}(c_1v_1 + v_1^2) = 526\frac{1}{2}.$$ (G)

[2] The expression in modern form is the equation: $5x^4 - 360x^3 - 1134x^2 + 321489 = 0$. (C)

[3] Here it means subtract from twice the area of Jia's segment the square of the *shi*, and divide the remainder by the *shi*. (C) The two characters *bei zhi* (double it) were lost, we add them according to the meaning of the text. (G)

[4] The expression in modern form is the equation: $-5x^4 + 360x^3 + 3240x^2 - 2624400 = 0$. (C)

[5] The two characters *bei zhi* (double) were lost, we add them according to the meaning of the text. That is, $c_2 = \frac{2(S_1 + S_2) - v_2^2}{v_2} = 72$. (G)

17.

【原文】

今有圆田一段，甲东截一弧，计积三十一步中半步；乙西截一弧，计积九十步。[1] 只云：甲截矢少如乙截矢三步[2]。问：二弧各截弦、矢几何？

答曰：甲截矢三步，截弦一十八步；

乙截矢六步，截弦二十四步。

术曰：立天元一为甲截矢，如积求之。得三万五千七百二十一为正实，三万五千七百二十一为从方，一万七百七十三为从上廉，九千六百六十六为益二廉，二百七为从三廉，三十三为从下廉，五为益隅，五乘方开之，[3] 得甲截矢。　又：立天元一为乙截矢，如积求之。得六十为益实，一十六为从方，一为益隅，平方开之，[4] 得乙截矢。合问。

【注释】

[1] 记甲、乙所截之弧田的面积、弦、矢分别为 S_1，S_2；c_1，v_1；c_2，v_2，此即：$S_1=\frac{1}{2}(c_1v_1+v_1^2)=31\frac{1}{2}$，$S_2=\frac{1}{2}(c_2v_2+v_2^2)=90$。（郭）

[2] 即：$v_2-v_1=3$。（郭）

[3] 开方式的现代形式为：

$-5x^6+33x^5+207x^4-9666x^3+10773x^2+35721x+35721=0$。（陈）

[4] 开方式的现代形式为：$-x^2+16x-60=0$。（陈）

17. Jia cut off a segment containing $31\frac{1}{2}$ *bu* from the east side of a circular tract of land and Yi a segment containing 90 *bu* from the west side.[1] It is known that the *shi* of Jia's segment is less than that of Yi's by 3 *bu*[2]. Find the *shi* and the *xian* of each segment.

Ans. Jia's *shi*, 3 *bu*;

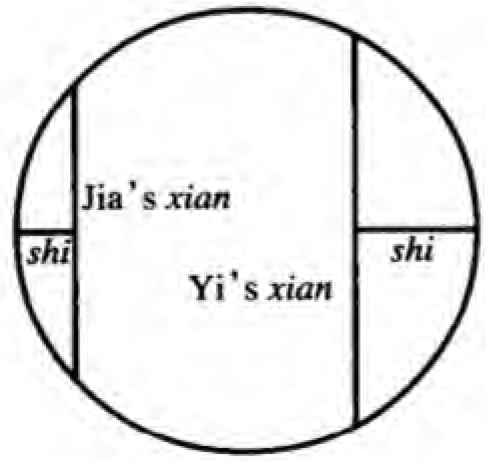
xian, 18 *bu*.

Yi's *shi*, 6 *bu*;

xian, 24 *bu*.

Process. Let the element *tian* be the *shi* of Jia's segment. From the statement we have 35721 for the positive *shi*, 35721 for the positive *fang*, 10773 for the positive upper *lian*, 9666 for the negative second *lian*, 207 for the positive third *lian*, 33 for the positive lower *lian*, and 5 for the negative *yu*, an expression[3] of the sixth degree whose root is the *shi* of Jia's segment.

Again let the element *tian* be the *shi* of Yi's segment. From the statement we have 60 for the negative *shi*, 16 for the positive *fang*, and 1 for the negative *yu*, a quadratic expression[4] whose root is the *shi* of Yi's segment.



【今译】

今有一段圆田，甲从东边截一弧田，计其面积为$31\frac{1}{2}$步，乙从西边截一弧田，计其面积为90步。只云甲截矢比乙截矢少3步。问：二弧田各截的弦、矢为多少？

答：甲截矢3步，截弦18步；

乙截矢6步，截弦24步。

术：设天元一为甲截矢，以如积方法求其解。得到35721为常数项，35721为一次项系数，10773为二次项系数，−9666为三次项系数，207为四次项系数，33为五次项系数，−5为最高次项系数，开六次方，得到甲截矢。　又：设天元一为乙截矢，以如积方法求其解。得到−60为常数项，16为一次项系数，−1为最高次项系数，开平方，得到乙截矢。符合所问。

18.

【原文】

今有大小圆田各一段，共地六亩六十四分亩之六十一。[1] 只云：小圆径如大圆径八分之五。今于二圆从边各截一弧，共积二百二十二步半。其小弧矢不及大弧矢三步。[2] 问：二弧矢各几何？

答曰：截大弧矢八步，弦三十二步；

截小弧矢五步，弦二十步。

术曰：立天元一为大圆径，如积求之。得一千六百为益实，一为从隅，平方开之，[3] 得大圆径四十步。五之，八而一，即小圆径。　又：立天元一为截小矢，如积求之。得四亿三千二百九十一万五千一百二十五为正实，四千二百四十三万八百为益方，一千二百六十万一千四百五十为益上廉，二十二万六千一百五十四为益二廉，一十八万七千五

[Notes]

[1] Let the area, the *xian*, and the *shi* of Jia's segment be S_1, c_1, and v_1, and the area, the *xian*, and the *shi* of Yi's segment be S_2, c_2, and v_2. From the statement we have

$$S_1 = \frac{1}{2}(c_1v_1 + v_1^2) = 31\frac{1}{2}, \quad S_2 = \frac{1}{2}(c_2v_2 + v_2^2) = 90. \text{ (G)}$$

[2] That is, $v_2 - v_1 = 3$. (G)

[3] The expression in modern form is the equation: $-5x^6 + 33x^5 + 207x^4 - 9666x^3 + 10773x^2 + 35721x + 35721 = 0$. (C)

[4] The expression in modern form is the equation: $-x^2 + 16x - 60 = 0$. (C)

18. The sum of the areas of two circular tracts of land, one large and the other small, is $6\frac{61}{64}$ *mu*.[1] The diameter of the small circle is only five-eighths that of the large. From each a segment is cut off the sum of their areas being $222\frac{1}{2}$ *bu*. The *shi* of the smaller segment is less by 3 *bu* than that of the large.[2] Find the *shi* of each segment.

Ans. The large circle:

shi, 8 *bu*;

xian, 32 *bu*.

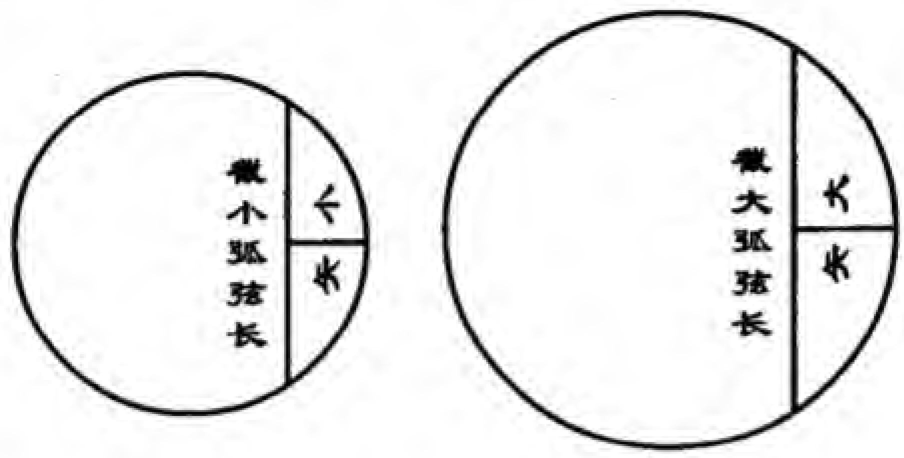
The small circle:

shi, 5 *bu*;

xian, 20 *bu*.

Process. Let the element *tian* be the diameter of the large circle. From the statement we have 1600 for the negative *shi*, and 1 for the positive *yu*, a

百一十一为从三廉，五千七百二为从四廉，七百五十三为益五廉，一十四为益下廉，一为正隅，七乘方开之，[4] 得截小矢五步。倍之，以减小圆径，余，自之，以减小圆径幂，余为实，开平方，即小弧弦。[5] 合问。



【注释】

[1] 记大、小圆的直径分别为 d_1，d_2，取 $\pi=3$，此即：$\frac{3}{4}d_1^2+\frac{3}{4}d_2^2=6\frac{61}{64}$ 亩 $=1668\frac{3}{4}$ 步，$d_2=\frac{5}{8}d_1$。（郭）

[2] 记从大、小圆所截之弧田的面积、弦、矢分别为 S_1，S_2；c_1，v_1；c_2，v_2，此即：$S_1+S_2=\frac{1}{2}(c_1v_1+v_1^2)+\frac{1}{2}(c_2v_2+v_2^2)=222\frac{1}{2}$。$v_2-v_1=3$。（郭）

[3] 开方式的现代形式为：$x^2-1600=0$。（陈）

[4] 开方式的现代形式为：

$$x^8-14x^7-753x^6+5702x^5+187511x^4-226154x^3-12601450x^2-42430800x+432915125=0$$ 。（陈）

[5] 此即：$c_2=\sqrt{d_2^2-(d_2-2v_2)^2}$，由《九章算术》勾股锯圆材术得来。（郭）

【今译】

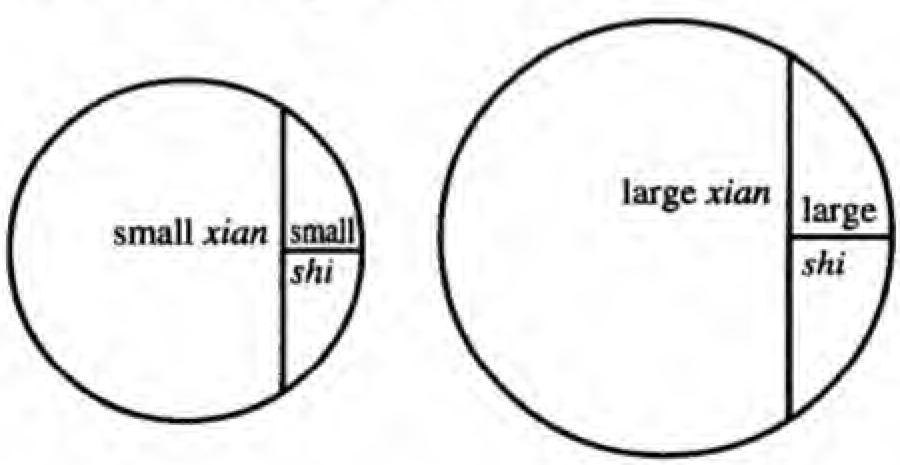
今有大、小各一段圆田，总共面积为 $6\frac{61}{64}$ 亩。只云：小圆径是大圆径的 $\frac{5}{8}$。现从二圆边上各截一弧田，其面积总共为 $222\frac{1}{2}$ 步。小弧田的矢比大弧田的矢少 3 步。问：二弧田的弦、矢各为多少？

答：截大弧矢 8 步，弦 32 步；

截小弧矢 5 步，弦 20 步。

quadratic expression [3] whose root, 40 *bu*, is the diameter of the large circle. Five-eighths of this diameter is the diameter of the small circle.

Again let the element *tian* be the *shi* of the small circle. From the statement we have 432915125 for the positive *shi*, 42430800 for the negative *fang*, 12601450 for the negative upper *lian*, 226154 for the negative second *lian*, 187511 for the positive third *lian*, 5702 for the positive fourth *lian*, 753 for the negative fifth *lian*, 14 for the negative lower *lian*, and 1 for the positive *yu*, an expression [4] of the eighth degree whose root, 5 *bu*, is the required *shi* of the small circle. Multiply the *shi* by 2. Subtract this value from the diameter of the small circle, square the result, and subtract it from the square of the diameter of the small circle. The square root of the remainder is the *xian* of the segment of the small circle [5].



[Notes]

[1] Let the diameter of the large circle be d_1, and the diameter of the small circle d_2. Using $\pi = 3$. From the statement we have $\frac{3}{4}d_1^2 + \frac{3}{4}d_2^2 = 6\frac{61}{64}$ mu $= 1668\frac{3}{4}$ bu, $d_2 = \frac{5}{8}d_1$. (G)

[2] Let the area, the *xian*, and the *shi* of the large circle's segment be S_1, c_1, and v_1, and the area, the *xian*, and the *shi* of the small circle's segment be S_2, c_2, and v_2. From the statement we have $S_1 + S_2 = \frac{1}{2}(c_1v_1 + v_1^2) + \frac{1}{2}(c_2v_2 + v_2^2) = 222\frac{1}{2}$, and $v_2 - v_1 = 3$. (G)

术：设天元一为大圆径，以如积方法求其解。得到 -1600 为常数项，1 为最高次项系数，开平方，得到大圆径 40 步。乘以 5，除以 8，即小圆径。 又：设天元一为截小弧矢，以如积方法求其解。得到 432915125 为常数项，-42430800 为一次项系数，-12601450 为二次项系数，-226154 为三次项系数，187511 为四次项系数，5702 为五次项系数，-753 为六次项系数，-14 为七次项系数，1 为最高次项系数，开八次方，得到截小弧矢 5 步。倍之，以减小圆径，余，自之，以减小圆径幂，余为实，开平方，就是小弧的弦。符合所问。

19.

【原文】

今有圆田一段，内复有圆池占之，二圆皆依古法，余地八亩强半亩。[1] 只云：环之实径自乘，多于通径二十步。今欲从西竖截车辋积五百三十八步。[2] 问：截池弦、池矢及内、外周、两头博径各几何？

答曰：截池矢六步，池弦三十六步；

内周三十七步二分，外周七十步四分；

博径一十四步，实径一十步。

术曰：立天元一为环之实径，如积求之。得七百为益实，二十为益方，一为益廉，一为从隅，立方开之，[3] 得实径。求得通径八十步，池径六十步。又二之辋积，以实径而一，得一百七步六分，为车辋内、外周相和之数。[4] 又：立天元一为截池矢，如积求之。得一兆四千九十九万七千九百一十七亿五千五百五十九万一千九百三十六为正实，二千五百五十八万三千亿三千九百八十三万八千七百二十为益方，一十八万七千八百二十九亿八千四百六十三万一十六为从上廉，二万七千六百五十四亿五千三百九十万九千七百六十为从二廉，一百二十四亿五千一百八十六万八千六十四为益三廉，九亿二千二百七十四万三千三百六十为益四廉，四百九十五万五千六百六十四为益五廉，八万二千三百二十为从下廉，二千四百一为从隅，七乘方开之，得截池矢六步。[5] 倍之，以减池径，余，自乘，复减池径幂，余为实，平方开之，得截池弦。[6] 又：池矢自乘，倍之，以池径除之，得数为池周弦差。加池弦得辋内周。以减内外周相和之数，余即外周[7]。

[3] The expression in modern form is the equation: $x^2 - 1600 = 0$. (C)

[4] The expression in modern form is the equation: $x^8 - 14x^7 - 753x^6 + 5702x^5 + 187511x^4 - 226154x^3 - 12601450x^2 - 42430800x + 432915125 = 0$. (C)

[5] That is, $c_2 = \sqrt{d_2^2 - (d_2 - 2v_2)^2}$. The result was from the *gou gu ju yuan* method in *The Nine Chapters of Mathematical Procedures*. (G)

19. A circular tract of land has a circular pond of water at its center. The area of the land is $8\frac{3}{4}$ *mu*.[1] The square of the *shi jing* exceeds the diameter by 20 *bu*. A piece of land in the form of a section of a wheel is cut from the west side, and its area is 538 (square) *bu*.[2] Find the *jie chi xian*, the *chi shi*, the interior and exterior arcs, and the *bo jing*.

Ans. *Jie chi shi*, 6 *bu*;

chi xian, 36 *bu*;

interior arc, 37 *bu* 2 *fen*;

exterior arc, 70 *bu* 4 *fen*;

bo jing, 14 *bu*;

shi jing, 10 *bu*.

Process. Let the element *tian* be the *shi jing*. From the statement we have 700 for the negative *shi*, 20 for the negative *fang*, 1 for the negative *lian*, and 1 for the positive *yu*, a cubic expression[3] whose root is the required *shi jing*. From this value we can easily obtain the diameter which is equal to 80 *bu*, and the diameter of the pond which is 60 *bu*. Twice the area of the tract of land in the form of a section of a wheel divided by the *shi jing* is

又：池矢加实径为通矢。[8] 自乘，倍之，以通径除之，所得为辋外周弦差。[9] 以减外周，余即通弦。[10] 内减池弦，余半之，即博径。[11] 合问。

【注释】

［1］记圆田、圆池周长分别为 l_1，l_2，此即：$\frac{1}{12}l_1^2-\frac{1}{12}l_2^2=8\frac{3}{4}$ 亩 = 2100 步。（郭）

［2］记圆田、圆环之径分别为 d_1，d，此即：$d^2-d_1=20$。记车辋田的外、内周分别为 L_1，L_2，则车辋田的面积为 $\frac{1}{2}(L_1+L_2)d=538$ 步。（郭）

［3］设天元一为环之实径，（郭）开方式的现代形式为：$x^3-x^2-20x-700=0$。（陈）

［4］"步"下之"六分"即0.6步。由以上开方式求得实径 $d=10$，则圆田之通径 $d_1=d^2-20=80$，车辋内外周之和 $L_1+L_2=\frac{2\times 538}{d}=107.6$ "以"，罗士琳改作"如"，无必要。"以"通"如"。（郭）

［5］截出车辋田时，亦在圆池内截出一弧田，称为截池。设天元一为截池矢。（郭）开方式的现代形式为：$2401x^8+82320x^7-4955664x^6-922743360x^5-12451688064x^4+2765453909760x^3+18782984630016x^2-2558300039838720x+14099791755591936=0$。（陈）

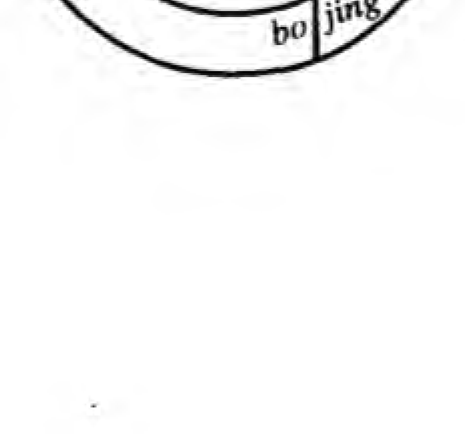
［6］记圆池的直径、截池的弦、矢分别为 d_2，c_2，v_2，由《九章算术》勾股锯圆材术得到：$c_2=\sqrt{d_2^2-(d_2-2v_2)^2}$。（郭）

［7］圆池的弧与弦之间的差。（陈）截池周即车辋田的内周 L_2，此由沈括的会圆术得到截池的周弦差 $L_2-c_2=\frac{2v_2^2}{d_2}$。由此求出车辋的内周，再由注［4］求出外周。（郭）

［8］外圆弧形的高。（陈）车辋田所在的从圆田截出的弧田的矢、弦称为通矢、通弦，记为 c_1，v_1，则 $v_1=v_2+d$。（郭）

107 *bu* 6 *fen*. The result equals the sum of the interior and exterior arcs.[4]

Again let the element *tian* be the *jie chi shi*. From the statement we have 14099791755591936 for the positive *shi*, 2558300039838720 for the negative *fang*, 18782984630016 for the positive upper *lian*, 2765453909760 for the positive second *lian*, 12451868064 for the negative upper third *lian*, 922743360 for the negative fourth *lian*, 4955664 for the negative fifth *lian*, 82320 for the positive lower *lian*, and 2401 for the positive *yu*, an expression of the eighth degree whose root is the required *jie chi shi* 6 *bu*.[5] Double this value and subtract from the diameter of the pond; square the result thus obtained and subtract from the square of the diameter of the pond the square root of the remainder is the *jie chi xian*.[6] Again twice the square of the *chi shi* divided by the diameter of the pond gives the *chi zhou xian cha*. Add to the *chi zhou xian cha* the *chi xian*, and we obtain the interior arc. From the sum of the interior and exterior arcs subtract the interior arc the remainder is the exterior arc.[7] Again, the sum of the *jie chi shi* and the *shi jing* is the *tong shi*.[8] Twice the square of the *tong shi* divided by the diameter is the *wang wai zhou xian cha*.[9] Subtract this quotient from the exterior arc, and the remainder is the *tong xian*.[10] Again, subtract from the *tong xian* the *jie chi xian*, and one-half of the remainder is the *bo jing*.[11]



[9] 外圆及其弦的差。(陈) 再由沈括的会圆术得到辋外的周弦差：$L_1 - c_1 = \frac{2v_1^2}{d_1}$。(郭)

[10] 外圆弧形的弦。(陈) $c_1 = L_1 - (L_1 - c_1)$。(郭)

[11] 博径为$\frac{1}{2}(c_1 - c_2)$。(郭)

【今译】

今有一段圆田，有一圆池占据中央，余下的田地面积为$8\frac{3}{4}$亩。只云：圆环的实径自乘比圆田的通径多20步。今欲从西边截出一车辋田，其面积为538步。问：所截的圆池的弦、矢，及车辋田的内外周、两头的博径各为多少？

答：所截圆池的矢6步，弦36步；

车辋田的内周37.2步，外周70.4步，博径14步；

环田的实径10步。

术：设天元一为圆环的实径，以如积方法求其解。得到-700为常数项，-20为一次项系数，-1为二次项系数，1为最高次项系数，开立方，得到实径。求得圆田的通径80步，圆池的直径60步。又：车辋田的面积乘以2，除以圆池的实径，得到107.6步为车辋田的内外周之和。　又：设天元一为所截圆池的矢，以如积方法求其解。得到14099791755591936为常数项，-2558300039838720为一次项系数，18782984630016为二次项系数，2765453909760为三次项系数，-12451868064为四次项系数，-9227443360为五次项系数，-4955664为六次项系数，82320为七次项系数，2401为最高次项系数，开八次方，得到所截圆池的矢6步。将所截圆池的矢加倍，以减圆池的直径，其余数自乘，以减圆池直径幂，余数作为常数项，开平方，就是所截圆池的弦。　又：所截圆池的矢自乘，加倍，除以圆池的直径，得到的数就是所截圆池周、弦之差。将此差加所截圆池的弦，便得到车辋田的内周。以内周减车辋田的内外周之和，余数就是外周。　又：所截圆池的矢加环田的实径，为所截圆田通矢。通矢自乘，加倍，以圆田的通径除之，所得就是车辋田的外周弦差。以外周弦差减外周，余数就是所截圆田的通弦。通弦内减去所截圆池的弦，取其余数的$\frac{1}{2}$，就是车辋田的博径。符合所问。

[Notes]

[1] Let the circumference of the circular tract of land be l_1, and the circumference of the circular pond l_2. From the statement we have $\frac{1}{12}l_1^2 - \frac{1}{12}l_2^2 = 8\frac{3}{4}$ mu = 2100 *bu*. (G)

[2] Let the diameter of the circular tract of land be d_1, and the *shi jing* d. From the statement we have $d^2 - d_1 = 20$. Let the interior and exterior arcs of the piece of land in the form of a section of a wheel be L_1 and L_2. Then, the land' s area is $\frac{1}{2}(L_1 + L_2)\,d = 538$ *bu*. (G)

[3] Let the element *tian* be the *shi jing*. (G) The expression in modern form is the equation: $x^3 - x^2 - 20x - 700 = 0$. (C)

[4] From the equation above, we have *shi jing* $d = 10$, the diameter of the circular tract of land is $d_1 = d_2 - 20 = 80$, and the sum of the interior and exterior arcs of the tract of land in the form of a section of a wheel $L_1 + L_2 = \frac{2 \times 538}{d} = 107\frac{6}{10}$. It is unnecessary that Luo Shilin changed character *yi* into *ru*. The character *yi* has a same meaning as *ru*. (G)

[5] When a piece of land in the form of a section of a wheel is cut, a piece of land in the form of a section of a circle in the pond is also cut. It is called *jie chi*. Let the element *tian* be the *jie chi shi*. (G) The expression in modern form is the equation: $2401x^8 + 82320x^7 - 4955664x^6 - 922743360x^5 - 12451688064x^4 + 2765453909760x^3 + 18782984630016x^2 - 2558300039838720x + 14099791755591936 = 0$. (C)

[6] Let the diameter of the circular pond be d_2, *jie chi xian* c_2, and *jie chi shi* v_2. From the *gou gu ju yuan cai* method in *The Nine Chapters of Mathematical Procedures*, we have $c_2 = \sqrt{d_2^2 - (d_2 - 2v_2)^2}$. (G)

[7] The difference between the arc and the chord of the circular pond.(C) The *jie chi zhou* is the interior arc of the land in the form of a section of a wheel L_2. From Shen Kuo' s *hui yuan* method, we have the *jie chi*'s *zhou xian cha* , that is, $L_2 - c_2 = \frac{2v_2^2}{d_2}$. From it we can have the interior arc of the tract of land in the form of a section of a wheel. And from note 4 we can have the exterior arc. (G)

[8] The altitude of the segment of the outer circle.(C) The *shi* and *xian* of the *hu tian* that is cut from the circular tract of land are called *tong shi* and *tong xian*. Use c_1 for *tong shi*, and v_1 for *tong xian*. Then, $v_1 = v_2 + d$. (G)

[9] The difference between the outer circle and its chord.(C) Again from Shen Kuo's *hui yuan* method, we have *zhou xian cha* of the outer circle of the land in the form of a section of a wheel. That is, $L_1 - c_1 = \frac{2v_1^2}{d_1}$. (G)

[10] The chord of the segment of the outer circle. (C) $c_1 = L_1 - (L_1 - c_1)$. (G)

[11] The *bo jing* is $\frac{1}{2}(c_1 - c_2)$. (G)

如像招数 五问

1.

【原文】

今有官司差夫一千八百六十四人筑堤。只云：初日差六十四人，次日转多七人。每人日支米三升，共支米四百三石九斗二升。[1]问：筑堤几日？

答曰：一十六日。

术曰：立天元一为茭草底子，如积求之。得一千八百为益实，六十七半为从方，三半为从隅，平方开之，[2]得茭草底子一十五束。加一即日数。 米求日者：立天元一为三角底子，如积求之。得八万四百为益实，五百九十为从方，二百一十三为从廉，七为从隅，立方开之，[3]得三角底子一十五个。加一即日数。合问。[4]

【注释】

[1] 记日数为n，以人数论，自第二日起是一个以$n-1$为底子的茭草垛，此即：$n\times 64+\frac{1}{2!}n(n-1)\times 7=1864$；以米数论，自第二日起是一个以$n-1$为底子的三角垛，此即$[\frac{1}{2!}n(n+1)\times 64+\frac{1}{3!}(n-1)n(n+1)\times 7]\times 3=40392$。（郭）

[2] 开方式的现代形式为：$3\frac{1}{2}x^2+67\frac{1}{2}x-1800=0$。（陈）

[3]“米求日者”，罗士琳改“者”作“术曰”，无必要。（郭）开方式的现代形式为：$7x^3+213x^2+590x-80400=0$。（陈）

[4] 此解题方法的现代形式如下：

$\frac{n[128+(n-1)7]}{2}=1864$或者$7n^2+121n-3728=0$，即$(n-16)(7n+233)=0$，所以$n=16$。（陈）

Ru Xiang Zhao Shu (Men Summoned According to Need)

5 Problems

1. The government called some workers to build a dike. On the first day there were 64 men but the number of laborers was increased each day by 7. The total number listed was 1864. The food paid to each laborer was 3 *sheng* per day and the total amount paid out was 403 *dan* 9 *dou* 2 *sheng*. [1] Find the number of days they spent on the work.

Ans. 16 days.

Process. Let the element *tian* be the base of the *jiao cao* pile (pile of hay). From the statement we have 1800 for the negative *shi*, $67\frac{1}{2}$ for the positive *fang*, and $3\frac{1}{2}$ for the positive *yu*, a quadratic expression[2] whose root, 15, is the required base. This value increased by 1 is the number of days spent on the work. Solving according to the given amount of rice we let the element *tian* be the base of the *san jiao* (triangular) pile. From the statement we have 80400 for the negative *shi*, 590 for the positive *fang*, 213 for the positive *lian*, and 7 for the positive *yu*, a cubic expression [3] whose root, 15, is the base of the *san jiao* pile. This value increased by 1 gives the number of days required. [4]

[Notes]

[1] Let the number of days be n. As far as the number of the workers is concerned, we have a *jiao cao* pile with $n - 1$ as a side of its base since the second day, that is,

$n \times 64 + \frac{1}{2!} n (n - 1) \times 7 = 1864$. As far as the amount of food is concerned, we have a *san jiao* pile with $n - 1$ as a side of its base since the second day, that is,

【今译】

今有官府差遣1864人去筑堤。只云：第一日派64人，自第二日起，每日多7人，每人每日支米3升，共支米403石9斗2升。问：筑堤日为多少?

答：16日。

术：设天元一为茭草底子，以如积方法求其解。得到-1800为常数项，$67\frac{1}{2}$为一次项系数，$3\frac{1}{2}$为最高次项系数，开平方，得到茭草底子15束。加1就是日数。 以米求日术：设天元一为三角底子，以如积方法求其解。得到-80400为常数项，590为一次项系数，213为二次项系数，7为最高次项系数，开立方，得到三角底子15个。加1，就是日数。符合所问。

2.

【原文】

今有官司依平方[1]招兵，初段方面四尺，次日方面转多二尺。[2]每人日给银一两二钱。已招兵四千九百五十六人，支银二万六千四十两。[3]问：招来几日?

答曰：一十四日。

术曰：立天元一为三角底子，如积求之。得七千三百五十六为益实，七十三为从方，二十一为从廉，二为从隅，立方开之，[4]得三角底子一十二个。加二即日数。 银求日术曰：立天元一为三角落一底子，如积求之。得六万四千八百九十六为益实，二百三十六为从方，九十五为从上廉，一十六为从下廉，一为从隅，三乘方开之，[5]得三角落一底子一十二个。加二即日数。合问。

【注释】

[1] 平方，为数列4^2，6^2，8^2，$10^2\cdots$。（陈）

$$\left[\frac{1}{2!}n(n+1) \times 64 + \frac{1}{3!}(n-1)n(n+1) \times 7\right] \times 3 = 40392.$$ (G)

[2] The expression in modern form is the equation: $3\frac{1}{2}x^2 + 67\frac{1}{2}x - 1800 = 0.$ (C)

[3] It is unnecessary that Luo Shilin changed the character *zhe* of *mi qiu ri zhe* into *shu yue*. (G) The expression in modern form is the equation: $7x^3 + 213x^2 + 590x - 80400 = 0.$ (C)

[4] The solution in modern form is as follows:

$\frac{n[128 + (n-1)7]}{2} = 1864$ or $7n^2 + 121n - 3728 = 0$. That is,

$(n - 16)(7n + 233) = 0$. Therefore, $n = 16$. (C)

2. The government summoned an army according to the *ping fang*[1] progression beginning with a square whose side was 4 the first day and increased it each day by 2.[2] The pay of each soldier was 1 *liang* 2 *qian* per day. The total number summoned was 4956 and the silver paid out was 26040 *liang*. [3] Find the number of days since the summons was sent out.

Ans. 14 days.

Process. Let the element *tian* be the base of the *san jiao* pile. From the statement we have 7356 for the negative *shi*, 73 for the positive *fang*, 21 for the positive *lian*, and 2 for the positive *yu*, a cubic expression[4] whose root is 12. This number increased by 2 is the required number of days since the summons was sent out. In solving from the amount paid out, let the element *tian* be *san jiao luo yi di zi*. From the statement we have 64896 for the negative *shi*, 236 for the positive *fang*, 95 for the positive upper *lian*, 16 for the positive lower *lian*, and 1 for the positive *yu*, an expression [5] of

[2] 据题意，初招之方面4尺，次日转多2尺，依平方招兵，此即：

差	Δ_1	Δ_2	Δ_3
初日方面4，按平方招：	$4^2 = 16$		
		20	
次日方面多2	$6^2 = 36$		8
		28	
三日方面多2	$8^2 = 64$		8
		36	
四日方面多2	$10^2 = 100$		

Δ_1，Δ_2，Δ_3分别是上差、中差、下差，下差皆相等，这是一个二阶招差问题。依据朱世杰的招差公式，n日共招兵数$f(n)$为

$$f(n) = n\Delta_1 + \frac{1}{2!}(n-1)n\Delta_2 + \frac{1}{3!}(n-2)(n-1)n\Delta_3$$

$$= n \times 16 + \frac{1}{2!}(n-1)n \times 20 + \frac{1}{3!}(n-2)(n-1)n \times 8 = 4956。\text{（郭）}$$

[3] “一两二钱”即1.2两。以银数论，记每日支银数为m，总银数为：

$$g(n) = \left[\frac{1}{2!}n(n+1)\Delta_1 + \frac{1}{3!}(n-1)n(n+1)\Delta_2 + \frac{1}{4!}(n-2)(n-1)n(n+1)\Delta_3\right]m = \left[\frac{1}{2!}n(n+1) \times 16 + \frac{1}{3!}(n-1)n(n+1) \times 20 + \frac{1}{4!}(n-2)(n-1)n(n+1) \times 8\right] \times 1.2 = 26040。\text{（郭）}$$

[4] 开方式的现代形式为：$2x^3 + 21x^2 + 73x - 7356 = 0$。（陈）

[5] 开方式的现代形式为：$x^4 + 16x^3 + 95x^2 + 236x - 64896 = 0$。（陈）

【今译】

今有官府要依据平方数招兵，第一日按照方面4尺，自第二日起每日多2尺。每人每日给银1两2钱。已招兵4956人，支银26040两。问：招兵日为多少？

答：14日。

the fourth degree whose root, 12, is the *san jiao luo yi di zi*. This root increased by 2 gives the required number of days.

【 Notes 】

[1] Square, the series, $4^2, 6^2, 8^2, 10^2 \cdots$. (C)

[2] According to the statement, we know that the side was 4 the first day and increased each day by 2 in a *ping fang* progression. Namely,

difference	Δ_1	Δ_2	Δ_3
the soldiers summoned according to the *ping fang* progression and the side on the first day was 4:			
	$4^2 = 16$		
		20	
the side increased by 2 on the second day:			
	$6^2 = 36$		8
		28	
the side increased by 2 on the third day:			
	$8^2 = 64$		8
		36	
the side increased by 2 on the fourth day:			
	$10^2 = 100$		

Δ_1, Δ_2, Δ_3 are the first difference, second difference, and the third difference respectively. All the third differences are equal. And this is a problem of the *er jie zhao cha* (method of finite or divided differences in two degrees). According to Zhu Shijie's formula of *zhao cha*, the number of soldiers summonded within *n* days is as follows:

术：设天元一为三角底子，以如积方法求其解。得到-7356为常数项，73为一次项系数，21为二次项系数，2为最高次项系数，开立方，得到三角底子12个。加2就是日数。　以银数求日数：设天元一为三角落一底子，以如积方法求其解。得到-64896为常数项，236为一次项系数，95为二次项系数，16为三次项系数，1为最高次项系数，开四次方，得到三角落一底子12个。加2，就是日数。符合所问。

3.

【原文】

今有官司依圆箭束招兵，初束外周一十二只，次束外周转多六只。[1]每人日给米四升。已招四千九百五人，支米九百三十一硕二斗。[2]问：招来几日？

答曰：一十五日。

术曰：立天元一为三角底子，如积求之。得四千八百四十九为益实，四十八为从方，一十二为从廉，一为从隅，立方开之，[3]得三角底子一十三个。加二即日数。　米求日术曰：立天元一为三角落一底子，如积求之。得九万二千八百二十为益实，三百二十八为从方，一百二十一为从上廉，一十八为从下廉，一为从隅，三乘方开之，[4]得三角落一底子一十三个。加二即日数。合问。

$$f(n) = n\Delta_1 + \frac{1}{2!}(n-1)n\Delta_2 + \frac{1}{3!}(n-2)(n-1)n\Delta_3$$
$$= n \times 16 + \frac{1}{2!}(n-1)n \times 20 + \frac{1}{3!}(n-2)(n-1)n \times 8 = 4956. \text{ (G)}$$

[3] 1 *liang* 2 *qian* is 1.2 *liang*. As far as the amount of silver was concerned, and m was the amount paid out every day, the total amount of silver was

$$g(n) = [\frac{1}{2!}n(n+1)\Delta_1 + \frac{1}{3!}(n-1)n(n+1)\Delta_2 + \frac{1}{4!}(n-2)(n-1)n(n+1)\Delta_3]m = [\frac{1}{2!}n(n+1) \times 16 + \frac{1}{3!}(n-1)n(n+1) \times 20 + \frac{1}{4!}(n-2)(n-1)n(n+1) \times 8] \times 1.2 = 26040. \text{ (G)}$$

[4] The expression in modern form is the equation: $2x^3 + 21x^2 + 73x - 7356 = 0$. (C)

[5] The expression in modern form is the equation: $x^4 + 16x^3 + 95x^2 + 236x - 64896 = 0$. (C)

3. The government summoned soldiers for public affairs according to the *yuan jian shu* progression. The circumference of the first round was 12 and each succeeding circumference was increased by 6.[1] Each soldier received for his ration 4 *sheng* of rice. 4905 soldiers were summoned and 931 *shuo* 2 *dou* of rice were paid out.[2] Find the number of days since the summons was sent out.

Ans. 15 days.

Process. Let the element *tian* be the base of the *san jiao* pile. From the statement we have 4849 for the negative *shi*, 48 for the positive *fang*, 12 for the positive *lian*, and 1 for the positive *yu*, a cubic expression[3] whose root, 13, is the required base. This root increased by 2 is the required number of days since the summons was sent out. In solving from the amount

【注释】

[1] 记圆箭外周、总数分别为为l, S，据杨辉《田亩比类乘除捷法》圆箭法：$S=\frac{1}{12}(l+6)l+1$。此题依圆箭束招兵，初日12只，次日起转多6只，此即：

差

		Δ_1	Δ_2	Δ_3
初日束12，按圆箭招：	$\frac{1}{12}(12+6)\times12+1=19$			
			18	
次日方面多6	$\frac{1}{12}(18+6)\times18+1=37$			6
			24	
三日方面多6	$\frac{1}{12}(24+6)\times24+1=61$			6
			30	
四日方面多6	$\frac{1}{12}(30+6)\times30+1=91$			

Δ_1，Δ_2，Δ_3分别是上差、中差、下差，下差皆相等，这也是一个二阶招差问题。依据朱世杰的招差公式，n日共招兵数$f(n)$为

$$f(n)=n\Delta_1+\frac{1}{2!}(n-1)n\Delta_2+\frac{1}{3!}(n-2)(n-1)n\Delta_3$$
$$=n\times19+\frac{1}{2!}(n-1)n\times18+\frac{1}{3!}(n-2)(n-1)n\times6=4905。$$（郭）

[2] 以米数论，记每日支米m，总米数为：

$$g(n)=\left[\frac{1}{2!}n(n+1)\Delta_1+\frac{1}{3!}(n-1)n(n+1)\Delta_2+\frac{1}{4!}(n-2)(n-1)n(n+1)\Delta_3\right]m=\left[\frac{1}{2!}n(n+1)\times19+\frac{1}{3!}(n-1)n(n+1)\times18+\frac{1}{4!}(n-2)(n-1)n(n+1)\times6\right]\times4=93120。$$（郭）

[3] 开方式的现代形式为：$x^3+12x^2+48x-4849=0$。（陈）

[4] 开方式的现代形式为：$x^4+18x^3+121x^2+328x-92820=0$。（陈）

【今译】

今有官府要依据圆箭束招兵，第一日按照外周12只，自第二日起每日多

of rations, let the element *tian* be the base of the *san jiao luo yi* pile. From the statement we have 92820 for the negative *shi*, 328 for the positive *fang*, 121 for the positive first *lian*, 18 for the positive lower *lian*, and 1 for the positive *yu*, an expression [4] of the fourth degree whose root, 13, is the required base. This value increased by 2 gives the number of days.

[Notes]

[1] Let the circumference and the total number of the *yuan jian* (round arrows-bundle) be l and S. From Yang Hui's *yuan jian* method in the *Tian Mu Bi Lei Cheng Chu Jie Fa*, we have $S = \frac{1}{12}(l + 6)l + 1$. The problem is concerned about soldiers summoned by *yuan jian shu*, and the circumference of the first round was 12 and each succeeding circumference was increased by 6. That is,

difference Δ_1 Δ_2 Δ_3

the first round was 12, and summoned soldiers according to *yuan jian shu*:

$$\frac{1}{12}(12 + 6) \times 12 + 1 = 19$$

18

the second round increased by 6:

$$\frac{1}{12}(18 + 6) \times 18 + 1 = 37 \qquad 6$$

24

the third round increased by 6:

$$\frac{1}{12}(24 + 6) \times 24 + 1 = 61 \qquad 6$$

30

the fourth round increased by 6:

$$\frac{1}{12}(30 + 6) \times 30 + 1 = 91$$

Δ_1, Δ_2, Δ_3 are the first difference, second difference, and third difference respectively. All the third differences are equal. And this is a problem of the *er jie zhao cha* (method of finite or divided differences in two degrees). According to Zhu Shijie's formula of *zhao cha*, the number of soldiers summoned within n days is

6只。每人每日给4升。已招兵4905人，支米931硕2斗。问：招兵日为多少？

答：15日。

术：设天元一为三角底子，以如积方法求其解。得到−4849为常数项，48为一次项系数，12为二次项系数，1为最高次项系数，开立方，得到三角底子13个。加2就是日数。　以米求日术：设天元一为三角落一底子，以如积方法求其解。得到−92820为常数项，328为一次项系数，121为二次项系数，18为三次项系数，1为最高次项系数，开四次方，得到三角落一底子13个。加2，就是日数。符合所问。

4.

【原文】

今有官司依平方招兵，初段方面五尺，次段方面转多一尺。[1]每人日给米三升，次日转多三升。已招二千四百四十人，支米四千四百七十七硕三斗二升。[2]问：招来几日？

答曰：一十五日。

术曰：立天元一为三角底子，如积求之。得一万四千二百七十四为益实，二百五十三为从方，三十九为从廉，二为从隅，立方开之，[3]得三角底子一十三个。加二即日数。　米求日术曰：立天元一为三角岚峰底子，如积求之。得五千三百六十七万四千九百二十为益实，七万三千三百八十六为从方，三万六千七百三十五为从上廉，七千九百五十为从二廉，七百五为从下廉，二十四为从隅，四乘方开之，[4]得三角岚峰底子一十三个。加二即日数。合问。

$$f(n) = n\Delta_1 + \frac{1}{2!}(n-1)n\Delta_2 + \frac{1}{3!}(n-2)(n-1)n\Delta_3$$
$$= n \times 19 + \frac{1}{2!}(n-1)n \times 18 + \frac{1}{3!}(n-2)(n-1)n \times 6 = 4905. \text{ (G)}$$

[2] As far as the amount of rice was concerned, and m was the amount rice paid out every day, the total amount of rice was

$$g(n) = [\frac{1}{2!}n(n+1)\Delta_1 + \frac{1}{3!}(n-1)n(n+1)\Delta_2 + \frac{1}{4!}(n-2)(n-1)n(n+1)\Delta_3]m = [\frac{1}{2!}n(n+1) \times 19 + \frac{1}{3!}(n-1)n(n+1) \times 18 + \frac{1}{4!}(n-2)(n-1)n(n+1) \times 6] \times 4 = 93120. \text{ (G)}$$

[3] The expression in modern form is the equation: $x^3 + 12x^2 + 48x - 4849 = 0$. (C)

[4] The expression in modern form is the equation: $x^4 + 18x^3 + 121x^2 + 328x - 92820 = 0$. (C)

4. The government summoned men for the army and they came each day according to the *ping fang* progression. On the first day a side of the square was 5. On each succeeding day the side was increased by 1.[1] Each soldier received 3 *sheng* of rice the first day and 3 additional *sheng* on each succeeding day. Finally 2440 men answered the summons and the rations paid out were 4477 *shuo* 3 *dou* 2 *sheng*.[2] Find the number of days.

Ans. 15 days.

Process. Let the element *tian* be the base of the *san jiao* pile. From the statement we have 14274 for the negative *shi*, 253 for the positive *fang*, 39 for the positive *lian*, and 2 for the positive *yu*, a cubic expression[3] whose root, 13, is the required base. This root increased by 2 gives the required number of days. For solving from the amount of rations given out, let the element *tian* be the base of the *san jiao lan feng* pile. From the statement we have 53674920 for the negative *shi*, 73386 for the positive *fang*, 36735

【注释】

［1］依平方招兵，先计算上、中、下三差Δ_1，Δ_2，Δ_3：

差	Δ_1	Δ_2	Δ_3
初日方面5，按平方招：	$5^2=25$		
		11	
次日方面多1	$6^2=36$		2
		13	
三日方面多1	$7^2=49$		2
		15	
四日方面多1	$8^2=64$		

Δ_1，Δ_2，Δ_3分别是25，11，2，这也是一个二阶招差问题。依据朱世杰的招差公式，n日共招兵数$f(n)$为

$$f(n)=n\Delta_1+\frac{1}{2!}(n-1)n\Delta_2+\frac{1}{3!}(n-2)(n-1)n\Delta_3$$

$$=n\times 25+\frac{1}{2!}(n-1)n\times 11+\frac{1}{3!}(n-2)(n-1)n\times 2=2440$$。（郭）

［2］以每日支米、次日转多数计，此当为岚峰形垛。上面计算的$5^2=25$，$6^2=36$，$7^2-6^2=13$，$(8^2-7^2)-(7^2-6^2)=2$，分别乘以1，2，3，4，得到25，72，39，8作为上差Δ_1，二差Δ_2，三差Δ_3，下差Δ_4，则依据朱世杰的招差公式，n日共支米数$g(n)$为$g(n)=[\frac{1}{2!}n(n+1)\Delta_1+\frac{1}{3!}(n-1)n(n+1)\Delta_2+\frac{1}{4!}(n-2)(n-1)n(n+1)\Delta_3+\frac{1}{5!}(n-3)(n-2)(n-1)n(n+1)\Delta_4]\times 3=149244\times 3=447732$。（郭）

［3］开方式的现代形式为：$2x^3+39x^2+253x-14274=0$。（陈）

［4］开方式的现代形式为：

$24x^5+705x^4+7950x^3+36735x^2+73386x-53674920=0$。（陈）

for the positive upper *lian*, 7950 for the positive second *lian*, 705 for the positive lower *lian*, and 24 for the positive *yu*, an expression[4] of the fifth degree whose root, 13, is the required base. This root increased by 2 gives the required number of days.

【Notes】

[1] According to the *ping fang* progression, one first calculates the first difference Δ_1, the second difference Δ_2, and the third difference Δ_3:

difference	Δ_1	Δ_2	Δ_3
the side was 5 the first day, and according to the *ping fang* progression:	$5^2 = 25$		
		11	
the side increased by 1 the second day			
	$6^2 = 36$		2
		13	
the side increased by 1 the third day			
	$7^2 = 49$		2
		15	
the side increased by 1 the fourth day 1			
	$8^2 = 64$		

Δ_1, Δ_2, Δ_3 are 25, 11, 2 respectiyely. This is also a problem of the *er jie zhao cha* (method of finite or divided differences in two degrees). According to Zhu Shijie's formula of *zhao cha*, the number of soldiers summoned within n days is

$$f(n) = n\Delta_1 + \frac{1}{2!}(n-1)n\Delta_2 + \frac{1}{3!}(n-2)(n-1)n\Delta_3$$
$$= n \times 25 + \frac{1}{2!}(n-1)n \times 11 + \frac{1}{3!}(n-2)(n-1)n \times 2 = 2440. \text{ (G)}$$

【今译】

今有官府要依据平方招兵，第一日按照方面5尺，自第二日起每日多1尺。每人每日给米3升，自第二日起每日增加3升。已招兵2440人，支米4477硕3斗2升。问：招兵日为多少？

答：15日。

术：设天元一为三角底子，以如积方法求其解。得到 −14274 为常数项，253 为一次项系数，39 为二次项系数，2 为最高次项系数，开立方，得到三角底子13个。加2就是日数。 以米数求日数术：设天元一为三角岚峰底子，以如积方法求其解。得到 −53674920 为常数项，73386 为一次项系数，36735 为二次项系数，7950 为三次项系数，705 为四次项系数，24 为最高次项系数，开五次方，得到三角岚峰底子13个。加2，就是日数。符合所问。

5.

【原文】

今有官司依立方招兵，初招方面三尺，次招方面转多一尺。每人日支钱二百五十文。已招二万三千四百人，支钱二万三千四百六十二贯。问：招来几日？

答曰：一十五日。

术曰：立天元一为三角落一底子，如积求之。得九万二千七百三十六为益实，六百六十为从方，一百八十一为从上廉，二十二为从下廉，一为正隅，三乘方开之，[1] 得三角落一底子一十二个。加三即日数。

钱求日术曰：立天元一为三角撒星底子，如积求之。得五百六十一万八百四十为益实，一万八千三百六十二为从方，六千三百九十为从

[2] According to the rice paid out every day and increased in each succeeding day, this is *lan feng* pile. Multiplying $5^2 = 25, 6^2 = 36, 7^2 - 6^2 = 13$, $(8^2 - 7^2) - (7^2 - 6^2) = 2$ by 1, 2, 3, 4 respectively, we have 25, 72, 39, 8 as the first difference Δ_1, the second difference Δ_2, the third difference Δ_3, and the fourth difference Δ_4. According to Zhu Shijie's formula of *zhao cha*, the total amount of rice within n days was as follows:

$$g(n) = [\frac{1}{2!}n(n+1)\Delta_1 + \frac{1}{3!}(n-1)n(n+1)\Delta_2 + \frac{1}{4!}(n-2)(n-1)n(n+1)\Delta_3 + \frac{1}{5!}(n-3)(n-2)(n-1)n(n+1)\Delta_4] \times 3 = 149244 \times 3 = 447732.$$ (G)

[3] The expression in modern form is the equation: $2x^3 + 39x^2 + 253x - 14274 = 0$. (C)

[4] The expression in modern form is the equation: $24x^5 + 705x^4 + 7950x^3 + 36735x^2 + 73386x - 53674920 = 0$. (C)

5. The government summoned men for the army according to the *li fang* progression beginning on the first day with a cube whose side was 3 *chi*. On each succeeding day the side was increased by 1. The pay of each soldier was 250 cash per day. The total number summoned was 23400 men and the money paid out was 23462000 cash. Find the number of days since the summons was sent out.

Ans. 15 days.

Process. Let the element *tian* be the base of the *san jiao luo yi* pile. From the statement we have 92736 for the negative *shi*, 660 for the positive *fang*, 181 for the positive upper *lian*, 22 for the positive lower *lian*, and 1 for the positive *yu*, an expression [1] of the fourth degree whose root, 12, is the

上廉，一千七十五为从二廉，九十为从三廉，三为正隅，四乘方开之，[2] 得三角撒星底子一十二个。加三即日数。或问：还原依立方招兵，初招方面三尺，次招方面转多一尺，得数为兵。今招一十五方。每人日支钱二百五十文。问：招兵及支钱各几何？ 答曰：兵二万三千四百人，钱二万三千四百六十二贯。 术曰：求得上差二十七，二差三十七，三差二十四，下差六。[3] 求兵者：今招为上积。又今招减一为茭草底子积，为二积。又今招减二为三角底子积，为三积。又今招减三为三角落一积，为下积。以各差乘各积，四位并之，即招兵数也。[4] 求支钱者：以今招为茭草积，为上积。又今招减一为三角底子积，为二积。又今招减二为三角落一积，为三积。又今招减三为三角撒星积，为下积。以各差乘各积，四位并之。所得，又以每日支钱乘之，即得支钱之数也。[5] 合问。

【注释】

[1] 开方式的现代形式为：$x^4+22x^3+181x^2+660x-92736=0$。（陈）

[2] 开方式的现代形式为：

$3x^5+90x^4+1075x^3+6390x^2+18362x-5610840=0$。（陈）

[3] 先计算上、二、三、下差 Δ_1，Δ_2，Δ_3，Δ_4：

差		Δ_1	Δ_2	Δ_3	Δ_4
初日方面3，按立方招：	$3^3=27$				
			37		
次日方面多1	$4^3=64$			24	
			61		6
三日方面多1	$5^3=125$			30	
			91		6
四日方面多1	$6^3=216$			36	
			127		6
五日方面多1	$7^3=343$			42	

Δ_1，Δ_2，Δ_3，Δ_4分别是27，37，24，6。四次差相等，故为四次招差。（郭）

required base. This root increased by 3 gives the required number of days.

In solving from the amount of money paid out, let the element *tian* be the base of the *san jiao sa xing* pile. From the statement we have 5610840 for the negative *shi*, 18362 for the positive *fang*, 6390 for the positive upper *lian*, 1075 for the positive second *lian*, 90 for the positive third *lian*, and 3 for the positive *yu*, an expression [2] of the fifth degree whose root, 12, is the required base. This root increased by 3 gives the required number of days. Note. Conversely: If the government summoned men for the army according to the *li fang* progression beginning on the first day with a cube whose side was 3 chi and increasing the side each day by 1 until it was 15 *chi*, the pay of each soldier being 250 cash per day, what was the total number of soldiers summoned and what the amount of money paid out?

Ans. Soldiers summoned, 23400;

money paid out, 23462000 cash.

Process. Find the first difference 27, the second difference 37, the third difference 24, and the fourth difference 6 [3]; then call the number, 15, which is a side of the last cube, the first sum (上积); the sum of the *jiao cao* pile whose base is 15 less 1, the second sum (二积); the sum of the *san jiao* pile, whose base is 15 less 2, the third sum (三积); and the sum of the *san jiao luo yi* pile, whose base is 15 less 3, the fourth sum (下积). After multiplying each sum by the corresponding difference, add the products together. The result is the number of soldiers summoned.[4] To find the amount of money paid out, take the number 15, which is a side of the last cube, for the first sum; the sum of the *jiao cao* pile, a side of whose base is 15 less 1, for the second sum; the sum of the *san jiao luo yi* pile, a side of whose base is 15 less 2, for the third sum; and the sum of the *san jiao sa xing* pile, a side of whose base is 15 less 3, for the fourth sum. After multiplying each sum by the corresponding difference, add their products and multiply by the amount paid out per day. The result is the total amount paid out [5].

【Notes】

[1] The expression in modern form is the equation: $x^4 + 22x^3 + 181x^2 + 660x -$

[4] 3^3, 4^3, 5^3, 6^3或者

27, 64, 125, 216

37, 61, 91

24 30

6

$d_1=27$, $d_2=37$, $d_3=24$, $d_4=6$。

15为上积，或S_1。

14为茭草垛底子，13为三角垛底子，12为三角落一形垛底子。

$S_1=15$, $S_2=\frac{14\cdot 15}{1\cdot 2}=105$, $S_3=\frac{13\cdot 14\cdot 15}{1\cdot 2\cdot 3}=455$,
$S_4=\frac{12\cdot 13\cdot 14\cdot 15}{1\cdot 2\cdot 3\cdot 4}=1365$。

$S_1d_1=15\times 27=405$

$S_2d_2=105\times 37=3885$

$S_3d_3=455\times 24=10920$

$S_4d_4=1365\times 6=8190$

23400,即解。(陈)

依据朱世杰的招差公式，n日共招兵数$f(n)$为

$$f(n)=n\Delta_1+\frac{1}{2!}(n-1)n\Delta_2+\frac{1}{3!}(n-2)(n-1)n\Delta_3+\frac{1}{4!}(n-3)(n-2)(n-1)n\Delta_4=n\times 27+\frac{1}{2!}(n-1)n\times 37+\frac{1}{3!}(n-2)(n-1)n\times 24+\frac{1}{4!}(n-3)(n-2)(n-1)n\times 6=23400。(郭)$$

[5] 记每日支钱为m，依据朱世杰的招差公式，n日共支钱数$g(n)$为

$$g(n)=[\frac{1}{2!}n(n+1)\Delta_1+\frac{1}{3!}(n-1)n(n+1)\Delta_2+\frac{1}{4!}(n-2)(n-1)n(n+1)\Delta_3+\frac{1}{5!}(n-3)(n-2)(n-1)n(n+1)\Delta_4]m=[\frac{1}{2!}n(n+1)\times 27+\frac{1}{3!}(n-1)n(n+1)\times 37+\frac{1}{4!}(n-2)(n-1)n(n+1)\times 24+\frac{1}{5!}(n-3)(n-2)(n-1)n(n+1)\times 6]\times 250=23462000。(郭)$$

92736 = 0. (C)

[2] The expression in modern form is the equation: $3x^5 + 90x^4 + 1075x^3 + 6390x^2 + 18362x - 5610840 = 0$ (C)

[3]First calculate the first difference Δ_1, the second difference Δ_2, the third difference Δ_3, and the fourth difference Δ_4:

difference	Δ_1	Δ_2	Δ_3	Δ_4
the side was 3 the first day, and according to the *li fang* progression:	$3^2 = 27$			
		37		
the side increased by 1 the second day:	$4^3 = 64$		24	
		61		6
the side increased by 1 the third day:	$5^3 = 125$		30	
		91		6
the side increased by 1 the fourth day:	$6^3 = 216$		36	
		127		6
the side increased by 1 the fifth day:	$7^3 = 343$		42	

Δ_1, Δ_2, Δ_3, Δ_4 are 27, 37, 24, 6 respectively. All the fourth differences are equal. So this is the *si ci zhao cha* problem. (G)

[4] 3^3, 4^3, 5^3, 6^3or

27, 64, 125, 216

37, 61, 91

24 30

6

$d_1 = 27, d_2 = 37, d_3 = 24, d_4 = 6$

【今译】

今有官府要依据立方招兵，第一日按照方面3尺，自第二日起每日多1尺。每人每日支钱250文。已招兵23400人，支钱23462贯。问：招兵日为多少？

答：15日。

术：设天元一为三角落一底子，以如积方法求其解。得到-92736为常数项，660为一次项系数，181为二次项系数，22为三次项系数，1为最高次项系数，开四次方，得到三角落一底子12个。加3就是日数。　以钱数求日数术：设天元一为三角撒星底子，以如积方法求其解。得到-5610840为常数项，18362为一次项系数，6390为二次项系数，1075为三次项系数，90为四次项系数，3为最高次项系数，开五次方，得到三角撒星底子12个。加3，就是日数。有人问：还原依照立方招兵，第一日按照方面3尺，自第二日起每日多1尺。其得数就是招兵数。现在招15方，每人每日支钱250文。问：招兵数及支钱各为多少？　答：招兵23400人，支钱23462贯。　术：先求得上差27，二差37，三差24，下差6。　求招兵数：现在招兵日数为上积。又现在招兵日数减1为茭草底子积，为二积。又现在招兵日数减2为三角底子积，为三积。又现在招兵日数减3为三角落一积，为下积。以各差乘各积，四位相加，就是招兵数。　求支钱数：以现在招兵日数为茭草积，为上积。又现在招兵日数减1为三角底子积，为二积。又现在招兵日数减2为三角落一积，为三积。又现在招兵日数减3为三角撒星积，为下积。以各差乘各积，四位相加。所得，再以每日支钱数乘之，就得到总支钱数。符合所问。

15 is the first sum or S_1

14 a side of the *jiao cao* pile,

13 a side of the *san jiao* pile,

and 12 a side of the *san jiao luo yi* pile.

$S_1 = 15, S_2 = \frac{14 \cdot 15}{1 \cdot 2} = 105, S_3 = \frac{13 \cdot 14 \cdot 15}{1 \cdot 2 \cdot 3} = 455,$

$S_4 = \frac{12 \cdot 13 \cdot 14 \cdot 15}{1 \cdot 2 \cdot 3 \cdot 4} = 1365.$

$S_1 d_1 = 15 \times 27 = 405$

$S_2 d_2 = 105 \times 37 = 3885$

$S_3 d_3 = 455 \times 24 = 10920$

$S_4 d_4 = 1365 \times 6 = 8190$

23400, Ans. (C)

According to Zhu Shijie' s formula of *zhao cha*, the number of soldiers summoned within n days was $f(n) = n\Delta_1 + \frac{1}{2!}(n-1)n\Delta_2 + \frac{1}{3!}(n-2)(n-1)n\Delta_3 + \frac{1}{4!}(n-3)(n-2)(n-1)n\Delta_4 = n \times 27 + \frac{1}{2!}(n-1)n \times 37 + \frac{1}{3!}(n-2)(n-1)n \times 24 + \frac{1}{4!}(n-3)(n-2)(n-1)n \times 6 = 23400.$ (G)

[5] Let the amount of money paid out every day be m. According to Zhu Shijie's formula of the method of finite or divided differences, the total amount mony paid out in n days was $g(n)$. And $g(n) = [\frac{1}{2!}n(n+1)\Delta_1 + \frac{1}{3!}(n-1)n(n+1)\Delta_2 + \frac{1}{4!}(n-2)(n-1)n(n+1)\Delta_3 + \frac{1}{5!}(n-3)(n-2)(n-1)n(n+1)\Delta_4]m = [\frac{1}{2!}n(n+1) \times 27 + \frac{1}{3!}(n-1)n(n+1) \times 37 + \frac{1}{4!}(n-2)(n-1)n(n+1) \times 24 + \frac{1}{5!}(n-3)(n-2)(n-1)n(n+1) \times 6] \times 250 = 23462000.$
(G)

四元玉鉴　卷下

果垛叠藏　二十问

1.

【原文】

今有三角垛果子一所，直钱一贯三百二十文。只云：从上一个直钱二文，次下层层每个累贵一文。[1] 问：底子每面几何？

答曰：九个。

术曰：立天元一为每面底子，如积求之。得三万一千六百八十为益实，十为从方，二十一为从上廉，一十四为从下廉，三为从隅，三乘方开之，[2] 得每面底子。合问。

【注释】

[1] 三角垛前 n 项之和是：$S_n=\sum_{i=1}^{n}\frac{1}{2!}i(i+1)=\frac{1}{3!}n(n+1)(n+2)$。此是三角垛的各项依次乘以某等差级数各项的求和问题。设等差级数首项为 a，公差为 d，则朱世杰使用的求和公式为 $S_n=\sum_{i=1}^{n}i(i+1)[a+(i+1)d]=\frac{1}{4!}n(n+1)(n+2)[3dn+(4a-3d)]$。此问中 $a=2$，$d=1$，则此 n 层三角垛的钱数是：$\frac{1}{4!}n(n+1)(n+2)[3\times1\times n+(4\times2-3\times1)]=1320$。（郭）

[2] 开方式的现代形式为：$3x^4+14x^3+21x^2+10x-31680=0$。（陈）

【今译】

今有一所三角垛果子，值钱1贯320文。只云上面一个值钱2文，以下一层比一层每个贵1文。问：底子的每面多少个？

答：9个。

BOOK Ⅲ
Guo Duo Die Cang (Piles of Fruit)
20 Problems

1. Apples are piled in the form of a triangular pyramid and the price of the whole is 1320 cash. The top apple is worth 2 cash. Each apple in a layer costs 1 cash less than an apple in the next layer below.[1] How many apples are there in the pile?

Ans. 9.

Process. Let the element *tian* be the number of apples in a side of the base. From the statement we have 31680 for the negative *shi*, 10 for the positive *fang*, 21 for the positive first *lian*, 14 for the positive last *lian*, and 3 for the positive *yu*, an expression [2] of the fourth degree whose root is the required number.

[Notes]

[1] The sum of the former n terms of the pile is $S_n = \sum_{i=1}^{n} \frac{1}{2!} i(i+1) = \frac{1}{3!} n(n+1)(n+2)$. It is a sum problem to multiply every term of a pile in the form of a triangular pyramid by every term of an arithmetic series in order. Let the first term of the arithmetic series be a, and difference d. The sum formula used by Zhu Shijie is as follows: $S_n = \sum_{i=1}^{n} i(i+1)[a+(i+1)d] = \frac{1}{4!} n(n+1)(n+2)[3dn+(4a-3d)]$. In this problem $a = 2$, and $d = 1$. Then the price of the whole for the pile with n layers is $\frac{1}{4!} n(n+1)(n+2)[3 \times 1 \times n + (4 \times 2 - 3 \times 1)] = 1320$. (G)

[2] The expression in modern form is the equation: $3x^4 + 14x^3 + 21x^2 + 10x - 31680 = 0$. (C)

术：设天元一为三角垛的底子的一面，以如积方法求其解。得到−31680为常数项，10为一次项系数，21为二次项系数，14为三次项系数，3为最高次项系数，开四次方，便得到每面底子。符合所问。

2.

【原文】

今有四角垛[1]果子一所，直钱一贯三百六十五文。只云：底子每个直钱一文，次上层层每个累贯二文。问：底子每面几何？

答曰：九个。

术曰：立天元一为每面底子，如积求之。得八千一百九十为益实，一为从方，二为从上廉，二为从下廉，一为正隅，三乘方开之。[2]合问。

【注释】

[1] 四角垛前n项之和是：$S_n=\sum_{i=1}^{n} i\times i=\frac{1}{3!}n(n+1)(2n+1)$。此是四角垛的各项依次乘以某等差级数各项的求和问题。设等差级数末项为b，公差为d，则朱世杰使用的求和公式为$S_n=\sum_{i=1}^{n} i^2[b-(n-i)]d)=\frac{1}{3!}n(n+1)[(2n+1)b-\frac{1}{2}(n-1)nd]$。此问中末项$b=1$，$d=-2$，则此$n$层四角垛的钱数是：$\frac{1}{3!}n(n+1)[(2n+1)\times 1-\frac{1}{2}(n-1)n\times(-2)]=1365$。（郭）

[2] 开方式的现代形式为：$x^4+2x^3+2x^2+x-8190=0$。（陈）

【今译】

今有一所四角垛果子，值钱1贯365文。只云底子的一个值钱1文，以上一层比一层每个贵2文。问：底子的每面多少个？

答：9个。

术：设天元一为四角垛每面的底子，以如积方法求其解。得到−8190

2. A pile of apples is in the shape of a pyramid[1] whose base is a square. The cost of the whole is 1365 cash. Each apple in the bottom layer is worth only 1 cash. Each apple in the next upper layer costs 2 cash more than those in the layer below. Find the number of apples in a side of the base.

Ans. 9.

Process. Let the element *tian* be the number of apples in a side of the base. From the statement we have 8190 for the negative *shi*, 1 for the positive *fang*, 2 for the positive first *lian*, 2 for the positive last *lian*, and 1 for the positive *yu*, an expression[2] of the fourth degree whose root is the required number.

[Notes]

[1] The sum of the former n terms of the pile is $S_n = \sum_{i=1}^{n} i \times i = \frac{1}{3!} n(n+1)(2n+1)$. It is a sum problem to multiply every term of a pile in the shape of a pyramid whose base is a square by every term of an arithmetic series in order. Let the last term of the arithmetic series be b, and difference d. The sum formula used by Zhu Shijie is as follows: $S_n = \sum_{i=1}^{n} i^2 [b - (n-i)] d) = \frac{1}{3!} n(n+1)[(2n+1)b - \frac{1}{2}(n-1)nd]$. In this problem, $b = 1$, *and* $d = -2$. Then the lost of the whole for the pile with n layers is $\frac{1}{3!} n(n+1)[(2n+1) \times 1 - \frac{1}{2}(n-1)n \times (-2)] = 1365$. (G)

[2] The expression in modern form is the equation $x^4 + 2x^3 + 2x^2 + x - 8190 = 0$. (C)

为常数项，1为一次项系数，2为二次项系数，2为三次项系数，1为最高次项系数，开四次方，便得到每面底子。符合所问。

3.

【原文】

今有四角落一形果子，积五百四十个。[1] 问：底子几何？

答曰：八个。

术曰：立天元一为四角落一底子，如积求之。得六千四百八十为益实，二为从方，五为从上廉，四为从下廉，一为正隅，三乘方开之。[2] 合问。

【注释】

[1] 此即：$S_n=\sum_{i=1}^{n}\frac{1}{3!}n(n+1)(2n+1)=\frac{1}{3!\times 2}(n+1)n(n+1)(n+2)=540$。（郭）

[2] 开方式的现代形式为：$x^4+4x^3+5x^2+2x-6480=0$。（陈）

【今译】

今有一所四角落一形垛果子，其积540个。问：底子的每面多少个？

答：8个。

术：设天元一为四角落一形垛每面的底子，以如积方法求其解。得到-6480为常数项，2为一次项系数，5为二次项系数，4为三次项系数，1为最高次项系数，开三次方，便得到每面底子。符合所问。

4.

【原文】

今有三角岚峰形果子，积六百三十个。[1] 问：底子几何？

答曰：六个。

3. The number of apples in a pile of a *si jiao luo yi* form is 540.[1] Find the number in a side of the base.

Ans. 8.

Process. Let the element *tian* be the number of apples in a side of the base. From the statement we have 6480 for the negative *shi*, 2 for the positive *fang*, 5 for the positive first *lian*, 4 for the positive last *lian*, and 1 for the positive *yu*, an expression [2] of the fourth degree whose root is the required number.

[Notes]

[1] That is, $S_n = \sum_{i=1}^{n} \frac{1}{3!} n(n+1)(2n+1) = \frac{1}{3! \times 2}(n+1)n(n+1)(n+2) = 540$. (G)

[2] The expression in modern form is the equation: $x^4 + 4x^3 + 5x^2 + 2x - 6480 = 0$. (C)

4. The number of apples in a pile of the *san jiao lan feng* form is 630.[1] Find a side of the base.

Ans. 6.

Process. Let the element *tian* be a side of the base. From the statement we

术曰：立天元一为三角岚峰底子，如积求之。得七万五千六百为益实，六为从方，三十五为从上廉，五十为从二廉，二十五为从三廉，四为从隅，四乘方开之。[2] 合问。

【注释】

[1] 此即：$\sum_{i=1}^{n}\frac{1}{3!}i(i+1)(i+2)\times i=\frac{1}{5!}n(n+1)(n+2)(n+3)(4n+1)=630$。（郭）

[2] 开方式的现代形式为：$4x^5+25x^4+50x^3+35x^2+6x-75600=0$。（陈）

【今译】

今有一所三角岚峰形垛果子，其积630个。问：底子的每面多少个？

答：6个。

术：设天元一为三角岚峰形垛每面的底子，以如积方法求其解。得到-75600为常数项，6为一次项系数，35为二次项系数，50为三次项系数，25为四次项系数，4为最高次项系数，开五次方，便得到每面底子。符合所问。

5.

【原文】

今有四角岚峰形果子，积四百四十八个。[1] 问：底子几何？

答曰：五个。

术曰：立天元一为四角岚峰底子，如积求之。得二万六千八百八十为益实，一为从方，一十二半为从上廉，二十五为从二廉，一十七半为从三廉，四为正隅，四乘方开之。[2] 合问。

have 75600 for the negative *shi*, 6 for the positive *fang*, 35 for the positive first *lian*, 50 for the positive second *lian*, 25 for the positive third *lian*, and 4 for the positive *yu*, an expression[2] of the fifth degree whose root is the required number.

[Notes]

[1] That is, $\sum_{i=1}^{n} \frac{1}{3!} i(i+1)(i+2) \times i = \frac{1}{5!} n(n+1)(n+2)(n+3)(4n+1) = 630$. (G)

[2] The expression in modern form is the equation: $4x^5 + 25x^4 + 50x^3 + 35x^2 + 6x - 75600 = 0$. (C)

5. The total number of apples in a *si jiao lan feng* pile is 448.[1] Find a side of the base.

Ans. 5.

Process. Let the element *tian* be a side of the base of the *si jiao lan feng* pile. From the statement we have 26880 for the negative *shi*, 1 for the positive *fang*, $12\frac{1}{2}$ for the positive first *lian*, 25 for the positive second *lian*, $17\frac{1}{2}$ for the positive third *lian*, and 4 for the positive *yu*, an expression[2] of

【注释】

[1] 此即：$\sum_{i=1}^{n}(\sum_{j=1}^{i}j^2)\cdot i=\sum_{i=1}^{n}\frac{1}{3!}i(i+1)(2i+1)\times i=\frac{1}{5!}n(n+1)(n+2)(8n^2+11n+1)=448$。（郭）

[2] 开方式的现代形式为：$4x^5+17\frac{1}{2}x^4+25x^3+12\frac{1}{2}x^2+x-26880=0$。（陈）

【今译】

今有一所四角岚峰形垛果子，其积448个。问：底子的每面多少个？

答：5个。

术：设天元一为四角岚峰形垛每面的底子，以如积方法求其解。得到-26880为常数项，1为一次项系数，$12\frac{1}{2}$为二次项系数，25为三次项系数，$17\frac{1}{2}$为四次项系数，4为最高次项系数，开五次方，便得到每面底子。符合所问。

6.

【原文】

今有三角撒星更落一形果子，积九百二十四个。[1] 问：底子几何？

答曰：七个。

术曰：立天元一为三角撒星更落一底子，如积求之。得六十六万五千二百八十为益实，一百二十为从方，二百七十四为从上廉，二百二十五为从二廉，八十五为从三廉，一十五为从四廉，一为正隅，五乘方开之。[2] 合问。

【注释】

[1] 此即：$\sum_{i=1}^{n}\frac{1}{5!}i(i+1)(i+2)(i+3)(i+4)=\frac{1}{6!}n(n+1)(n+2)(n+3)(n+4)(n+5)=924$。（郭）

the fifth degree whose root is the required number.

【 Notes 】

[1] That is, $\sum_{i=1}^{n}(\sum_{j=1}^{i} j^2)\cdot i = \sum_{i=1}^{n}\frac{1}{3!}i(i+1)(2i+1)\times i = \frac{1}{5!}n(n+1)(n+2)(8n^2+11n+1) = 448$. (G)

[2] The expression in modern form is the equation: $4x^5 + 17\frac{1}{2}x^4 + 25x^3 + 12\frac{1}{2}x^2 + x - 26880 = 0$. (C)

6. The total number of apples in a *san jiao sa xing geng luo yi* pile is 924.[1] Find a side of the base.

Ans. 7.

Process. Let the element *tian* be a side of the base of the *san jiao sa xing geng luo yi* pile. From the statement we have 665280 for the negative *shi*, 120 for the positive *fang*, 274 for the positive first *lian*, 225 for the positive second *lian*, 85 for the positive third *lian*, 15 for the positive fourth *lian*, and 1 for the positive *yu*, an expression[2] of the sixth degree whose root is the required number.

【 Notes 】

[1] That is, $\sum_{i=1}^{n}\frac{1}{5!}i(i+1)(i+2)(i+3)(i+4) = \frac{1}{6!}n(n+1)(n+2)$

［2］开方式的现代形式为：$x^6+15x^5+85x^4+225x^3+274x^2+120x-665280=0$。（陈）

【今译】

今有一所三角撒星更落一形垛果子，其积924个。问：底子的每面多少个？

答：7个。

术：设天元一为三角撒星更落一形垛每面的底子，以如积方法求其解。得到－665280为常数项，120为一次项系数，274为二次项系数，225为三次项系数，85为四次项系数，15为五次项系数，1为最高次项系数，开六次方，便得到每面底子。符合所问。

7.

【原文】

今有奇层圆锥垛果子，积九百三十二个。[1] 问：高几层？

答曰：一十五层。

术曰：立天元一为层数，如积求之。得七千四百五十五为益实，二为从方，三为从廉，二为从隅，立方开之。[2] 合问。

【注释】

［1］记层数为n，此即：$\frac{1}{8}[n(n+1)(2n+1)+(n+1)]=932$。（郭）

［2］开方式的现代形式为：$2x^3+3x^2+2x-7455=0$。（陈）

【今译】

今有一所奇数层圆锥垛果子，其积932个。问：高是几层？

$(n+3)(n+4)(n+5) = 924$. (G)

[2] The expression in modern form is the equation: $x^6 + 15x^5 + 85x^4 + 225x^3 + 274x^2 + 120x - 665280 = 0$. (C)

7. The total number of apples in a pile in the form of a cone is 932, and the number of layers is an odd number.[1] Find the number of layers.

Ans. 15.

Process. Let the element *tian* be the number of layers. From the statement we have 7455 for the negative *shi*, 2 for the positive *fang*, 3 for the positive first *lian*, and 2 for the positive *yu*, a cubic expression[2] whose root is the required number.

[Notes]

[1] Let the number of layers be n, that is, $\frac{1}{8}[n(n+1)(2n+1)+(n+1)] = 932$. (G)

[2] The expression in modern form is the equation: $2x^3 + 3x^2 + 2x - 7455 = 0$. (C)

答：15层。

术：设天元一为奇层圆锥垛的层数，以如积方法求其解。得到 −7455 为常数项，2为一次项系数，3为二次项系数，2为最高次项系数，开立方，便得到层数。符合所问。

8.

【原文】

今有三角台垛果子，积五百四个。只云上下面底子和得二十一个。[1]问：上下面各几何？

答曰：上面七个，下面一十四个。

术曰：立天元一为下面底子，如积求之。得六千一百三十二为益实，六百六十二为从方，二十为益廉，一为正隅，立方开之。[2] 合问。

【注释】

[1] 记上、下面底子分别为为 m、n，此即：$\frac{1}{6}[n(n+1)(n+2)+(m-1)m(m+1)]=504$，$m+n=21$。（郭）

[2] 开方式的现代形式为：$x^3-30x^2+662x-6132=0$。（陈）

【今译】

今有一所三角台垛果子，其积504个。只云上、下面底子数之和为21个问：上下底的一面各多少？

答：上底一面7个，下底一面14个。

术：设天元一为下底一面的个数，以如积方法求其解。得到 −6132 为常数项，662为一次项系数，30为二次项系数，1为最高次项系数，开立方，便得到个数。符合所问。

8. A pile of apples in the form of a truncated regular triangular pyramid has 21 apples in the side of its bases.[1] The total number of apples is 504. Find the sides of the bases.

Ans. Side of the upper base, 7;

side of the lower base, 14.

Process. Let the element *tian* be a side of the lower base. From the statement we have 6132 for the negative *shi*, 662 for the positive *fang*, 30 for the negative *lian*, and 1 for the positive *yu*, a cubic expression [2] whose root is the required number.

[Notes]

[1] Let the sides of the pile's upper and lower bases be m and n. From the statement we have $\frac{1}{6}[n(n+1)(n+2)+(m-1)m(m+1)]=504, m+n=21$. (G)

[2] The expression in modern form is the equation: $x^3-30x^2+662x-6132=0$. (C)

9.

【原文】

今有四角台垛果子，积一千一百一十一个。只云上面不及下面五个，却多层数五个。[1] 问：上下面及高各几何？

答曰：上面一十一个，下面一十六个，高六层。

术曰：立天元一为上面个数，如积求之。得六千九百四十一为益实，九十五为益方，六为从隅，立方开之。[2] 合问。

【注释】

[1] 记上、下面与高分别为 m, n, h，此即：

$$\frac{1}{6}\{[(2n+1)+m]n+[(2m-1)+n]m\}h=1111,$$

$$n-m=5,\ m-h=5。(郭)$$

[2] 开方式的现代形式为：$6x^3-95x-6941=0$。(陈)

【今译】

今有一所四角台垛果子，其积1111个。只云：上底的一面比下底的一面少5个，却比层数多5。问：上下底的一面及高各多少？

答：上底一面11个，下底一面16个，高6层。

术：设天元一为上底一面的个数，以如积方法求其解。得到−6941为常数项，−95为一次项系数，6为最高次项系数，开立方，便得到上底一面的个数。符合所问。

10.

【原文】

今有刍童垛果子，积八十二个。[1] 只云：并下长、上阔，平方开之，加

9. A pile of apples in the form of a truncated pyramid with a square base has its total number equal to 1111. A side of the upper base contains 5 less than a side of the lower base. But the number of layers of the upper base is 5 more than that of the lower base.[1] Find the number in the sides of the bases and in the altitude of the pile.

Ans. Side of the upper base, 11;

side of the lower base, 16;

altitude, 6.

Process. Let the element *tian* be the number of apples in the upper base. From the statement we have 6941 for the negative *shi*, 95 for the negative *fang*, and 6 for the positive *yu*, a cubic expression[2] whose root is the required number.

[Notes]

[1] Let the pile's side of the upper base, side of the lower base, and altitude be m, n, and h. From the statement we have $\frac{1}{6}\{[(2n+1)+m]n+[(2m-1)+n]m\}h=1111, n-m=5, m-h=5$. (G)

[2] The expression in modern form is the equation $6x^3-95x-6941=0$. (C)

10. The number of apples in a *chu tong* pile is 82.[1] The square root of the number in the length of the lower base plus the widths of the upper and lower bases is 8. The lower width contains 2 less than the lower length, the upper

入下阔，共得八个。下阔不及下长二个，上阔如上长二分之一，高与上长同。[2] 问：上、下长、阔及高各几何？

答曰：下阔五个，下长七个；高四；

上阔二个，上长四个。

术曰：立天元一为下阔，如积求之。得九十五万三千一百九十为正实，七十七万二千三百六十八为益方，二十五万四千八百六十一为从上廉，四万三千七百三十八为益二廉，四千一百一十二为从三廉，二百一为益四廉，四为正隅，五乘方开之。[3] 合问。

【注释】

[1] 记刍童垛的上下底的长、阔及高分别为 a_1，b_1，a_2，b_2，h，根据北宋沈括《梦溪笔谈》提出的刍童垛的求积公式，有 $S=\frac{1}{6}[(2b_1+b_2)a_1+(2b_2+b_1)a_2+(b_2-b_1)]h=82$。（郭）

[2] 此即：$\sqrt{b_2+a_1}+a_2=8, b_2-a_2=2, a_1=\frac{1}{2}b_1, b_1=h$。（郭）

[3] 开方式的现代形式为：$4x^6-201x^5+4112x^4-43738x^3+254861x^2-772368x+953190=0$。（陈）

【今译】

今有一所刍童垛果子，其积82个。只云：下长加上阔，开平方，加下阔，共得8个。下阔比下长少2个。上阔是上长的$\frac{1}{2}$，高与上长相等。问：上、下长、阔及高各几何？

答：下阔5个，下长7个；高4层；

上阔2个，上长4个。

术：设天元一为刍童垛的下阔，以如积方法求其解。得到953190为常数项，−772368为一次项系数，254861为二次项系数，−43738为三次项系数，4112为四次项系数，−201为五次项系数，4为最高次项系数，开六次方，便得到下阔的个数。符合所问。

width contains one-half as many as the upper length, and the height contains the same number as the upper length.[2] Find the number in the widths and the lengths of the bases and the height of the pile.

Ans. Lower width, 5;

lower length, 7;

height, 4;

upper width, 2;

upper length, 4.

Process. Let the element *tian* be the number of apples in the lower width. From the statement we have 953190 for the positive *shi*, 772368 for the negative *fang*, 254861 for the positive first *lian*, 43738 for the negative second *lian*, 4112 for the positive third *lian*, 201 for the negative fourth *lian*, and 4 for the positive *yu*, an expression[3] of the sixth degree whose root is the required number.

[Notes]

[1] Let the *chu tong* pile's upper length be b_1, upper width a_1, lower length b_2, lower width a_2, and height h. From the formula of the volume of the *chu tong* pile provided by Shen Kuo in his *Meng Xi Bi Tan*, we have $S = \frac{1}{6}[(2b_1 + b_2)a_1 + (2b_2 + b_1)a_2 + (b_2 - b_1)]h = 82$. (G)

[2] That is, $\sqrt{b_2 + a_1} + a_2 = 8$, $b_2 - a_2 = 2$, $a_1 = \frac{1}{2}b_1$, $b_1 = h$. (G)

[3] The expression in modern form is the equation: $4x^6 - 201x^5 + 4112x^4 - 43738x^3 + 254861x^2 - 772368x + 953190 = 0$. (C)

11.

【原文】

今有刍甍垛果子，积一百个。[1]只云：并下长、下阔及高为共，减二，余，以平方开之，与上长等。下长多于上长中半。上长不及下阔一个。[2]问：上、下长、阔及高各几何？

答曰：上长四个，下长八个；

下阔五个，高五个。

术曰：立天元一为上长，如积求之。得一百二十为益实，二为从方，五为益上廉，一为益下廉，一为正隅，三乘方开之。[3]合问。

【注释】

[1] 记刍甍垛的上下底的长、阔及高分别为 b_1, a_2, b_2, h，此即：

$S=\frac{1}{6}(2b_2+b_1)(a_2+1)h=100$。（郭）

[2] 此即：$\sqrt{(b_2+a_2+h)-2}=b_1$，$b_2-b_1=\frac{1}{2}b_2$，$a_2-b_1=1$。（郭）

[3] 开方式的现代形式为：$x^4-x^3-5x^2+2x-120=0$。（陈）

【今译】

今有一所刍甍垛果子，其积100个。只云：下长、下阔与高相加，减二，其余数开平方，得数与上长相等。下长比上长多下长的$\frac{1}{2}$。上长比下阔少1个。问：上、下长、阔及高各几何？

答：上长4个，下长8个；

下阔5个；高5个。

术：设天元一为刍甍垛的上长，以如积方法求其解。得到−120为常数项，2为一次项系数，−5为二次项系数，−1为三次项系数，1为最高次项系数，开四次方，便得到上长的个数。符合所问。

11. The number of apples in a chu meng pile is 100.[1] It is said that the square root of the sum of the apples in the lower length, the lower width, and the height, decreased by 2, is the same as the number in the upper length; the number in the lower length exceeds that in the upper by one-half of itself; and the number in the upper length is less than that in the lower width by 1.[2] Find the upper and lower lengths, the width, and the height of the pile.

Ans. Upper length, 4;

lower length, 8;

lower width, 5;

height, 5.

Process. Let the element *tian* be the upper length. From the statement we have 120 for the negative *shi*, 2 for the positive *fang*, 5 for the negative first *lian*, 1 for the negative last *lian*, and 1 for the positive *yu*, an expression[3] of the fourth degree whose root is the required length.

【Notes】

[1] Let the *chu meng* pile's upper length be b_1, lower width a_2, lower length b_2, and height h. From the statement we have $S = \frac{1}{6}(2b_2 + b_1)(a_2 + 1)h = 100$. (G)

[2] That is, $\sqrt{(b_2 + a_2 + h)} - 2 = b_1, b_2 - b_1 = \frac{1}{2}b_2, a_2 - b_1 = 1$. (G)

[3] The expression in modern form is the equation: $x^4 - x^3 - 5x^2 + 2x - 120 = 0$. (C)

12.

【原文】

今有圆锥垛果子一所，令甲、乙、丙分之。甲分五百八个，乙分四百一个，丙分二百一十五个。从上给丙，奇层；次中给乙，偶层；次下与甲，奇层。[1] 问：各分层数几何？

答曰：甲三层，乙四层，丙九层。

术曰：立天元一为丙分层数，如积求之。得一千七百一十九为益实，二为从方，三为从廉，二为从隅，立方开之，[2] 得丙分层数。又立天元一为乙丙共分层数，如积求之。得四千九百二十七为益实，二为从方，三为从廉，二为从隅，立方开之，[3] 得一十三层。内减丙分层数，余即乙分层数。又立天元一为共高层数，如积求之。得八千九百九十二为益实，二为从方，三为从廉，二为从隅，立方开之，[4] 得共高一十六层，内减乙、丙层数，余即甲分层数。合问。

【注释】

[1] 由甲、丙分奇层，乙分偶层，知此圆锥垛为偶数层，记层数为 n，其积为 $508+401+215=1124$，因此，$\frac{1}{8}[n(n+1)(2n+1)+n]=1124$。记丙分层数，丙、乙共分层数分别为 n_1, n_2，由于 n_1, n_2 都是奇数，则 $\frac{1}{8}[n_1(n_1+1)(2n_1+1)+(n_1+1)]=215$，$\frac{1}{8}[n_2(n_2+1)(2n_2+1)+(n_2+1)]=215+401=616$。（郭）

[2] 开方式的现代形式为：$2x^3+3x^2+2x-1719=0$。（陈）

[3] 开方式的现代形式为：$2x^3+3x^2+2x-4927=0$。（陈）

[4] 开方式的现代形式为：$2x^3+3x^2+2x-8992=0$。（陈）

12. A pile of apples in the form of a circular cone is divided among Jia, Yi, and Bing. Jia receives 508, Yi 401, and Bing 215. Bing's apples are from the top layers, odd in number; Yi's from the middle layers, even in number; and Jia' s from the lower layers, odd in number. [1] Find the number of layers each received.

Ans. Jia, 3 layers;

Yi, 4 layers;

Bing, 9 layers.

Process. Let the element *tian* be the number of layers which Bing received. From the statement we have 1719 for the negative *shi*, 2 for the positive *fang*, 3 for the positive *lian*, and 2 for the positive *yu*, a cubic expression[2] whose root is the number of layers Bing received. Again let the element *tian* be the number of layers which Yi and Bing together received. From the statement we have 4927 for the negative *shi*, 2 for the positive fang, 3 for the positive *lian*, and 2 for the positive *yu*, a cubic expression[3] whose root is 13. From this root subtract the number of layers which Bing received, and we shall have then the number of layers Yi received. Again let the element *tian* be the whole number of layers in the pile. From the statement we have 8992 for the negative *shi*, 2 for the positive *fang*, 3 for the positive *lian*, and 2 for the positive *yu*, a cubic expression [4] whose root is 16. Subtract from this root the number of layers Yi and Bing received, and we shall then have the number Jia received.

[Notes]

[1] As Jia and Bing recesived odd layers, and Yi received even layers, we know

【今译】

今有一所圆锥垛果子，令甲、乙、丙三人分之：甲分508个，乙分401个，丙分215个；从上而下，先给丙，是奇数层，中间给乙，是偶数层，下面给甲，是奇数层。问：各人分的层数是多少？

答：甲3层，乙4层，丙9层。

术：设天元一为丙分层数，以如积方法求其解。得到 -1719 为常数项，2为一次项系数，3为二次项系数，2为最高次项系数，开立方，便得到丙分层数。　又：设天元一为乙丙共分层数，以如积方法求其解。得到 -4927 为常数项，2为一次项系数，3为二次项系数，2为最高次项系数，开立方，得到13层，内中减去丙分层数，余数就是乙分层数。　又：设天元一为三人共同的层数，以如积方法求其解。得到 -8992 为常数项，2为一次项系数，3为二次项系数，2为最高次项系数，开立方，得到16层，内中减去乙、丙的层数，余数就是甲分层数。符合所问。

13.

【原文】

今有四角垛果子一所，令甲、乙、丙分之。甲分五百九十个，乙分四百四十六个，丙分二百四个。从下给甲，次中与乙，次上与丙。[1] 问：各分层数几何？

答曰：甲三层，乙四层，丙八层。

术曰：立天元一为共高层数，如积求之。得七千四百四十为益实，一为从方，三为从廉，二为从隅，立方开之，[2] 得共高层数。　又：立天元一为丙分层数，如积求之。得一千二百二十四为益实，一为从方，三为从廉，二为从隅，立方开之，[3] 得丙分八层。　又立天元一为乙

that the number of the layers of the pile is even. Let the number of the layers be n. Its volume is 508 + 401 + 215 = 1124. Therefore, $\frac{1}{8}[n(n+1)(2n+1)+n] = 1124$. Let the number of layers divided by Bing be n_1, the number of layers divided by Bing and Yi n_2, and because n_1 and n_2 are odd numbers, we have $\frac{1}{8}[n_1(n_1+1)(2n_1+1)+(n_1+1)] = 215$, $\frac{1}{8}[n_2(n_2+1)(2n_2+1)+(n_2+1)] = 215+401 = 616$. (G)

[2] The expression in modern form is the equation: $2x^3 + 3x^2 + 2x - 1719 = 0$. (C)

[3] The expression in modern form is the equation: $2x^3 + 3x^2 + 2x - 4927 = 0$. (C)

[4] The expression in modern form is the equation: $2x^3 + 3x^2 + 2x - 8992 = 0$. (C)

13. A pile of apples in the form of a pyramid with a square base is divided among Jia, Yi, and Bing. Jia receives 590, Yi 446, and Bing 204. Jia's layers are from the lower layers of the pile, Yi's from the middle, and Bing's from the top. [1] How many layers does each receive from the pile?

Ans. Jia, 3 layers;

Yi, 4 layers;

Bing, 8 layers.

Process. Let the element *tian* be the number of layers in the whole pile. From the statement we have 7440 for the negative *shi*, 1 for the positive *fang*, 3 for the positive *lian*, and 2 for the positive *yu*, a cubic expression [2] whose root

丙共分层数，如积求之。得三千九百为益实，一为从方，三为从廉，二为从隅，立方开之，[4]得一十二层。内减丙分层数，余为乙分层数。以减共高，余即甲分层数。合问。

【注释】

[1] 记此四角垛的层数为 n，其积为 $590+446+204=1240$，因此 $\frac{1}{3!}n(n+1)(2n+1)=1240$。记丙分层数，丙、乙共分层数分别为 n_1，n_2，则 $\frac{1}{3!}n_1(n_1+1)(2n_1+1)=204$，$\frac{1}{3!}n_2(n_2+1)(2n_2+1)=650$。(郭)

[2] 开方式的现代形式为：$2x^3+3x^2+x-7440=0$。(陈)

[3] 开方式的现代形式为：$2x^3+3x^2+x-1224=0$。(陈)

[4] 开方式的现代形式为：$2x^3+3x^2+x-3900=0$。(陈)

【今译】

今有一所四角垛果子，令甲、乙、丙三人分之：甲分 590 个，乙分 446 个，丙分 204 个；从下而上，先给甲，中间给乙，上面给丙。问：各人分的层数是多少？

答：甲 3 层，乙 4 层，丙 8 层。

术：设天元一为共高层数，以如积方法求其解。得到 −7440 为常数项，1 为一次项系数，3 为二次项系数，2 为最高次项系数，开立方，便得到共高层数。又：设天元一为丙分层数，以如积方法求其解。得到 −1224 为常数项，1 为一次项系数，3 为二次项系数，2 为最高次项系数，开立方，得到丙分 8 层。又：设天元一为乙丙共分层数，以如积方法求其解。得到 −3900 为常数项，1 为一次项系数，3 为二次项系数，2 为最高次项系数，开立方，得到 12 层。内中减去丙分层数，余数就是乙分层数。以乙丙共分层数减共高层数，余数就是甲分层数。符合所问。

is the number of layers. Again let the element *tian* be the number of layers which Bing receives. From the statement we have 1224 for the negative *shi*, 1 for the positive *fang*, 3 for the positive *lian*, and 2 for the positive *yu*, a cubic expression [3] whose root is the number of layers Bing receives.

Again let the element *tian* be the number of layers which Yi and Bing receive. From the statement we have 3900 for the negative *shi*, 1 for the positive *fang*, 3 for the positive *lian*, and 2 for the positive *yu*, a cubic expression [4] whose root, 12, is the number of layers received by Yi and Bing. From this number subtract the number of layers Bing receives. The result is the number received by Yi. Subtract the root from the number of layers in the whole pile. The result is the number of layers Jia receives.

【Notes】

[1] Let the number of layers of the pile be n. Its volume is $590 + 446 + 204 = 1240$. Thus, $\frac{1}{3!} n(n+1)(2n+1) = 1240$. Let the number of layers divided by Bing be n_1, and the number of layers divided by Bing and Yi n_2. Then, $\frac{1}{3!} n_1(n_1+1)(2n_1+1) = 204$, $\frac{1}{3!} n_2(n_2+1)(2n_2+1) = 650$. (G)

[2] The expression in modern form is the equation: $2x^3 + 3x^2 + x - 7440 = 0$. (C)

[3] The expression in modern form is the equation: $2x^3 + 3x^2 + x - 1224 = 0$. (C)

[4] The expression in modern form is the equation: $2x^3 + 3x^2 + x - 3900 = 0$. (C)

14.

【原文】

今有三角、四角垛果子各一所，共积一百一十一个。[1] 只云四角底面不及三角底面一个。[2] 问：二底面各几何?

答曰：三角底面六个，四角底面五个。

术曰：立天元一为三角底面，如积求之。得二百二十二为益实，一为从方，一为从隅，立方开之，得三角底面。[3] 合问。

【注释】

[1] 记三角垛、四角垛的底一边个数分别是n_1,n_2，此即：

$$\frac{1}{3!}n_1(n_1+1)(n_1+2)+\frac{1}{3!}n_2(n_2+1)(2n_2+1)=111$$。(郭)

[2] 此即：$n_1-n_2=1$。(郭)

[3] 开方式的现代形式为：$x^3+x-222=0$。(陈)

【今译】

今各有一所三角垛、四角垛果子，共积111个。只云四角垛底的一边个数比三角垛底的一边个数少1个。问：二垛底的一边个数各是多少?

答：三角垛底的一边为6个，四角垛底的一边为5个。

术：设天元一为三角垛底的一边个数，以如积方法求其解。得到-222为常数项，1为一次项系数，1为最高次项系数，开立方，便得到三角垛底的一边个数。符合所问。

15.

【原文】

今有三角、四角垛果子各一所，四角积内减三角积，余二十个。[1] 只云

14. The number of apples in two piles, one in the form of a pyramid whose base is an equilateral triangle and the other in the form of a pyramid whose base is a square, is 111.[1] It is said that a side of the square base is less by 1 than a side of the triangular base.[2] Find a side of each of the bases.

Ans. A side of the triangular base, 6;

a side of the square base, 5.

Process. Let the element *tian* be a side of the triangular base. From the statement we have 222 for the negative *shi*, 1 for the positive *fang*, and 1 for the positive *yu*, a cubic expression[3] whose root is the required side.

[Notes]

[1] Let a side of the triangular base be n_1, and a side of the square base n_2. From the statement we have $\frac{1}{3!}n_1(n_1+1)(n_1+2)+\frac{1}{3!}n_2(n_2+1)(2n_2+1)=111$. (G)

[2] That is, $n_1-n_2=1$. (G)

[3] The expression in modern form is the equation: $x^3+x-222=0$. (C)

15. The difference between the number of apples in two piles in the form of pyramids, one with a square base and the other a triangular, is 20.[1] The sum of the apples in a side of the square base and a side of the triangular is 15.[2]

三角、四角底面和得一十五个。[2] 问：各几何？

答曰：四角底面七个，三角底面八个。

术曰：立天元一为四角底子，如积求之。得一千四百为益实，二百五十六为从方，一十五为益廉，一为正隅，立方开之。[3] 合问。

【注释】

[1] 记三角垛、四角垛的底一边个数分别是 n_1，n_2，此即：

$\frac{1}{3!}n_2(n_2+1)(2n_2+1)-\frac{1}{3!}n_1(n_1+1)(n_1+2)=20$。(郭)

[2] 此即：$n_1+n_2=15$。(郭)

[3] 开方式的现代形式为：$x^3-15x^2+256x-1400=0$。(陈)

【今译】

今各有一所三角垛、四角垛果子，四角垛的积减去三角垛的积，余20个。只云三角垛与四角垛底的一边之和是15个。问：二垛底的一边个数各是多少？

答：四角垛底的一边为7个，三角垛底的一边为8个。

术：设天元一为四角垛底的一边个数，以如积方法求其解。得到−1400为常数项，256为一次项系数，−15为二次项系数，1为最高次项系数，开立方，便得到四角垛底一边的个数。符合所问。

16.

【原文】

今有三角垛果子三所、四角垛果子六所，共积一千二百七十二个。[1] 只云四角底面乘三角底面得四十八个。[2] 问：二底面各几何？

答曰：四角底面四个，三角底面一十二个。

术曰：立天元一为四角底子，如积求之。得五万五千二百九十六为正

How many apples are there in a side of each of the base?

Ans. Square base, 7;

triangular base, 8.

Process. Let the element *tian* be the number of apples in a side of the square base. From the statement we have 1400 for the negative *shi*, 256 for the positive *fang*, 15 for the negative *lian*, and 1 for the positive *yu*, a cubic expression[3] whose root is the required number.

[Notes]

[1] Let a side of the triangular base be n_1, and a side of the square base n_2. From the statement we have $\frac{1}{3!} n_2 (n_2 + 1)(2n_2 + 1) - \frac{1}{3!} n_1 (n_1 + 1)(n_1 + 2) = 20$. (G)

[2] That is, $n_1 + n_2 = 15$. (G)

[3] The expression in modern form is the equation: $x^3 - 15x^2 + 256x - 1400 = 0$. (C)

16. There are three equal piles of apples whose bases are equilateral triangles and six equal piles whose bases are squares. The total number of apples is 1272.[1] It is said that the product of the number in a side of the square base by the number in a side of the triangular base is equal to 48.[2] Find the number in a side of each of the bases.

Ans. Square base, 4;

实，三千四百五十六为从方，四十八为从上廉，一千二百七十二为益二廉，一为从三廉，三为从下廉，二为从隅，五乘方开之。[3] 合问。

【注释】

[1] 记三角垛、四角垛的底一边个数分别是 n_1, n_2，此即：

$3\times\frac{1}{3!}n_1(n_1+1)(n_1+2)+6\times\frac{1}{3!}n_2(n_2+1)(2n_2+1)=1272$。(郭)

[2] 此即：$n_1n_2=48$。(郭)

[3] 开方式的现代形式为：$2x^6+3x^5+x^4-1272x^3+48x^2+3456x+55296=0$。(陈)

【今译】

今各有3所三角垛果子、6所四角垛果子，共积1272个。只云四角垛与三角垛底的一边相乘得到48个。问：二垛底的一边个数各是多少？

答：四角垛底的一边为4个，三角垛底的一边为12个。

术：设天元一为四角垛底的一边个数，以如积方法求其解。得到55296为常数项，3456为一次项系数，48为二次项系数，−1272为三次项系数，1为四次项系数，3为五次项系数，2为最高次项系数，开六次方，便得到四角垛底一边的个数。符合所问。

17.

【原文】

今有三角垛果子二所、四角垛果子三所，共积六百五十二个。[1] 只云三角底面除四角底面得二个。[2] 问：二底面各几何？

答曰：三角底面四个，四角底面八个。

术曰：立天元一为三角底子，如积求之。得一千九百五十六为益实，

triangular base, 12.

Process. Let the element *tian* be the number of apples in a side of the square base. From the statement we have 55296 for the positive *shi*, 3456 for the positive *fang*, 48 for the positive first *lian*, 1272 for the negative second *lian*, 1 for the positive third *lian*, 3 for the positive last *lian*, and 2 for the positive *yu*, an expression[3] of the sixth degree whose root is the required number.

[Notes]

[1] Let a side of the triangular base be n_1, and a side of the square base n_2. From the statement we have $3 \times \frac{1}{3!} n_1 (n_1 + 1)(n_1 + 2) + 6 \times \frac{1}{3!} n_2 (n_2 + 1)(2n_2 + 1) = 1272$. (G)

[2] That is, $n_1 n_2 = 48$. (G)

[3] The expression in modern form is the equation: $2x^6 + 3x^5 + x^4 - 1272x^3 + 48x^2 + 3456x + 55296 = 0$. (C)

17. The number of apples in two equilateral pyramids with triangular bases and three equivalent pyramids with square bases is 652.[1] It is said that the quotient of the number in a side of the square base by the number in a side of the triangular base is 2.[2] Find the number of apples in a side of each of the bases.

Ans. Triangular base, 4;

五为从方，二十一为从上廉，二十五为从隅，立方开之。[3] 合问。

【注释】

[1] 记三角垛、四角垛的底一边个数分别是 n_1，n_2，此即：

$$2\times\frac{1}{3!}n_1(n_1+1)(n_1+2)+3\times\frac{1}{3!}n_2(n_2+1)(2n_2+1)=652。（郭）$$

[2] 此即：$\frac{n_2}{n_1}=2$。（郭）

[3] 开方式的现代形式为：$25x^3+21x^2+5x-1956=0$。（陈）

【今译】

今各有2所三角垛果子、3所四角垛果子，共积652个。只云四角垛底的一边除以三角垛底的一边得2个。问：二垛底的一边个数各是多少？

答：三角垛底的一边为4个，四角垛底的一边为8个。

术：设天元一为三角垛底的一边个数，以如积方法求其解。得到 -1956为常数项，5为一次项系数，21为二次项系数，25为最高次项系数，开立方，便得到三角垛底一边的个数。符合所问。

18.

【原文】

今有四角垛果子积，以三角垛果子积除之，得七个。[1] 只云三角底面如四角底面七分之四。[2] 问：二底面各几何？

答曰：三角底面四个，四角底面七个。

术曰：立天元一为三角底子，如积求之。得三百九十二为正实，三百七十八为从方，一百一十九为益隅，平方开之。[3] 合问。

square base, 8.

Process. Let the element *tian* be the number of apples in a side of the triangular base. From the statement we have 1956 for the negative *shi*, 5 for the positive *fang*, 21 for the positive first *lian*, and 25 for the positive *yu*, a cubic expression [3] whose root is the required number of apples in a side of the triangular base.

【Notes】

[1] Let a side of the triangular base be n_1, and a side of the square base n_2. From the statement we have $2 \times \frac{1}{3!} n_1(n_1+1)(n_1+2) + 3 \times \frac{1}{3!} n_2(n_2+1)(2n_2+1) = 652$. (G)

[2] That is, $\frac{n_2}{n_1} = 2$. (G)

[3] The expression in modern form is the equation: $25x^3 + 21x^2 + 5x - 1956 = 0$. (C)

18. The quotient of the number of apples in a pyramid with a square base and the number in a pyramid with an equilateral triangle as its base is 7.[1] The number in a side of the triangular base is four-sevenths of the number in a side of the square base.[2] Find the number in a side of each of the bases.

Ans. Triangular base, 4;

square base, 7.

Process. Let the element *tian* be the number of apples in a side of the triangular base. From the statement we have 392 for the positive *shi*, 378 for the

【注释】

[1] 记三角垛、四角垛的底一边个数分别是 n_1,n_2,此即:

$$\frac{\frac{1}{3!}n_2(n_2+1)(2n_2+1)}{\frac{1}{3!}n_1(n_1+1)(n_1+2)}=7。(郭)$$

[2] 此即:$n_1=\frac{4}{7}n_2$。(郭)

[3] 开方式的现代形式为:$-119x^2+378x+392=0$。(陈)

【今译】

今有四角垛果子积,除以三角垛果子积,得7。只云三角垛底的一边的个数是四角垛底的一边个数的 $\frac{4}{7}$ 。问:二垛底的一边个数各是多少?

答:三角垛底的一边为4个,四角垛底的一边为7个。

术:设天元一为三角垛底的一边个数,以如积方法求其解。得到392为常数项,378为一次项系数,-119为最高次项系数,开平方,便得到三角垛底一边的个数。符合所问。

19.

【原文】

今有三角、四角果子积,相乘得二万三千一百个。[1] 只云并三角、四角底面,平方开之,不及四角底面三个。[2] 问:二底面各几何?

答曰:四角底面七个,三角底面九个。

术曰:立天元一为四角底面,如积求之。得八十三万一千六百为益实,九百九十为从方,八百七十七为从上廉,二千五百三十为益二廉,三百五十八为从三廉,一千四百二十六为从四廉,一千一十六为益五廉,二百九十二为从六廉,三十九为益下廉,二为从隅,八乘方开之。[3] 合问。

positive fang, and 119 for the negative *yu*, a quadratic expression[3] whose root is the required number.

[Notes]

[1] Let a side of the triangular base be n_1, and a side of the square base n_2. From the statement we have $\dfrac{\frac{1}{3!}n_2(n_2+1)(2n_2+1)}{\frac{1}{3!}n_1(n_1+1)(n_1+2)}=7$. (G)

[2] That is, $n_1=\frac{4}{7}n_2$. (G)

[3] The expression in modern form is the equation: $-119x^2+378x+392=0$. (C)

19. The product of the number of apples in the form of a pyramid whose base is an equilateral triangle by the number in the form of a pyramid whose base is square, is 23100.[1] It is said that the square root of the number in a side of the triangular base plus the number in a side of the square base is less than the number in a side of the square base by 3.[2] Find the number in a side of each of the bases.

Ans. Square base, 7;

triangular base, 9.

Process. Let the element *tian* be the number of apples in a side of the square base. From the statement we have 831600 for the negative *shi*, 990 for the

【注释】

[1] 记三角垛、四角垛的底一边个数分别是 n_1, n_2, 此即:

$$\left[\frac{1}{3!}n_1(n_1+1)(n_1+2)\right]\left[\frac{1}{3!}n_2(n_2+1)(2n_2+1)\right]=23100$$。(郭)

[2] 此即: $\sqrt{n_1+n_2}=n_2-3$。(郭)

[3] 开方式的现代形式为: $2x^9-39x^8+292x^7-1016x^6+1426x^5+358x^4-2530x^3+877x^2+990x-831600=0$。(陈)

【今译】

今有三角垛果子积与四角垛果子积相乘，得到23100个。只云三角垛与四角垛的一边相加，开平方，得数比四角垛底的一边少3个。问：二垛底的一边个数各是多少?

答：四角垛底的一边为7个，三角垛底的一边为9个。

术：设天元一为四角垛底的一边个数，以如积方法求其解。得到−831600为常数项，990为一次项系数，877为二次项系数，−2530为三次项系数，358为四次项系数，1426为五次项系数，−1016为六次项系数，292为七次项系数，−39为八次项系数，2为最高次项系数，开九次方，便得到四角垛底一边的个数。符合所问。

20.

【原文】

今有三角、四角果子各一所，共积二百一十一个。[1] 只云三角底子一层之数与四角底子一层之数等。问：二底面各几何?

答曰：三角底面八个，四角底面六个。

术曰：立天元一为三角底面，如积求之。得六百四十一万一千二十四为正实，三万五千四百五十为益方，四万五千五百三十三为益上廉，一万一十二为益二廉，九十九为从三廉，三十为从下廉，二为正隅，

positive *fang*, 877 for the positive first *lian*, 2530 for the negative second *lian*, 358 for the positive third *lian*, 1426 for the positive fourth *lian*, 1016 for the negative fifth *lian*, 292 for the positive sixth *lian*, 39 for the negative last *lian*, and 2 for the positive *yu*, an expression[3] of the ninth degree whose root is the number of apples in a side of the square base.

【 Notes 】

[1] Let a side of the triangular base be n_1, and a side of the square base n_2. From the statement we have $[\frac{1}{3!}n_1(n_1+1)(n_1+2)][\frac{1}{3!}n_2(n_2+1)(2n_2+1)] = 23100$. (G)

[2] That is, $\sqrt{n_1+n_2} = n_2 - 3$. (G)

[3] The expression in modern form is the equation: $2x^9 - 39x^8 + 292x^7 - 1016x^6 + 1426x^5 + 358x^4 - 2530x^3 + 877x^2 + 990x - 831600 = 0$. (C)

20. The number of apples in two pyramid piles, the base of one being an equilateral triangle and the other a square, is 211.[1] It is said that the number of apples in the perimeters of the first layers of these two piles is the same. Find the number in a side of each of the bases.

Ans. Triangular base, 8;

square base, 6.

Process. Let the element *tian* be the number of apples in a side of the triangular base. From the statement we have 6411024 for the positive *shi*, 35450

五乘方开之。[2] 合问。

【注释】

[1] 记三角垛、四角垛的底一边个数分别是 n_1, n_2，此即：

$\frac{1}{3!}n_1(n_1+1)(n_1+2)+\frac{1}{3!}n_2(n_2+1)(2n_2+1)=211$。(郭)

[2] 开方式的现代形式为：$2x^6+30x^5+99x^4-10012x^3-45533x^2-35450x+6411024=0$。(陈)

【今译】

今有三角垛果子、四角垛果子各有一所，共积211个。只云三角底子的一层之数与四角底子的一层之数相等。问：二垛底的一边个数各是多少?

答：三角垛底的一边为8个，四角垛底的一边为6个。

术：设天元一为三角垛底的一边个数，以如积方法求其解。得到6411024为常数项，−35450为一次项系数，−45533为二次项系数，−10012为三次项系数，99为四次项系数，30为五次项系数，2为最高次项系数，开六次方，便得到三角垛底一边的个数。符合所问。

for the negative *fang*, 45533 for the negative first *lian*, 10012 for the negative second *lian*, 99 for the positive third *lian*, 30 for the positive last *lian*, and 2 for the positive *yu*, an expression[2] of the sixth degree whose root is the number of apples in a side of the triangular base.

[Notes]

[1] Let a side of the triangular base be n_1, and a side of the square base n_2. From the statement we have $\frac{1}{3!}n_1(n_1+1)(n_1+2)+\frac{1}{3!}n_2(n_2+1)(2n_2+1)=211$. (G)

[2] The expression in modern form is the equation: $2x^6+30x^5+99x^4-10012x^3-45533x^2-35450x+6411024=0$. (C)

锁套吞容　一十九问

1.

【原文】

今有圆田一段，内有圆池占之，余积六百一十二步。[1] 只云实径自乘，不及内周四十八步，却与内、外周差等。[2] 问：三事各几何？

答曰：实径六步，内周八十四步，外周一百二十步。

术曰：立天元一为实径，如积求之。得二千四百四十八为益实，三十二为从上廉，一为从隅，三乘方开之，[3] 得实径。合问。

【注释】

[1] 取 $\pi=3$，记圆田、圆池的周长即其内、外周分别为 l_1，l_2，此即：$\frac{1}{12}l_1^2-\frac{1}{12}l_2^2=612$。（郭）

[2] 记实径为 d，此即：$l_2-d^2=48$，$d^2=l_1-l_2$。（郭）

[3] 开方式的现代形式为：$x^4+32x^2-2448=0$。（陈）

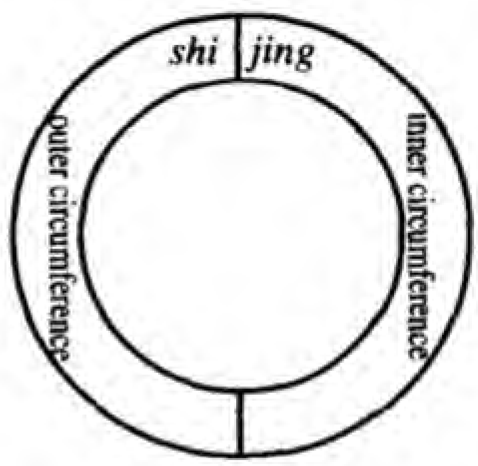
【今译】

今有一段圆田，一段圆池占据中央，剩余的面积为612步。只云：环田的实径自乘，比内周少48步，而与内、外周之差相等。问：实径、内周、外周各为多少？

Suo Tao Tun Rong (Figures within Other Figures)

19 Problems

1. In the center of a circular piece of land is a circular pond of water. The area of the land is 612 (square) *bu*.[1] It is said that the square of the *shi jing* is less than the inner circumference by 48 *bu*, but is equal to the difference between the outer and the inner circumferences.[2] Find the three matters.



Ans. *Shi jing*, 6 *bu*;

inner circumference, 84 *bu*;

outer circumference, 120 *bu*.

Process. Let the element *tian* be the *shi jing*. From the statement we have 2448 for the negative *shi*, 32 for the positive first *lian*, and 1 for the positive *yu*, an expression [3] of the fourth degree whose root is the required *shi jing*.

[Notes]

[1] Make $\pi = 3$, and let the circumferences of the circular piece of land and circular pond, that is, the land's outer circumference and the inner circumference of the land,

答：实径6步，内周84步，外周120步。

术：设天元一为实径，以如积方法求其解。得到 −2448 为常数项，32为二次项系数，1为最高次项系数，开四次方，得到实径。符合所问。

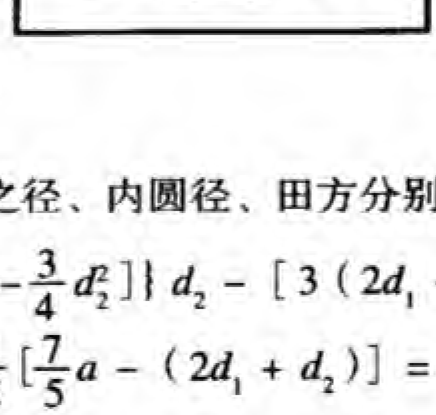
2.

【原文】

今有方田一段，内有环池占之。余积以环内圆径乘之，减外周幂，余二万五千一百六十四步。[1] 只云四角至池外楞各长一十一步半，内、外周差三十六步。[2] 问：三事各几何？

答曰：内圆径二十八步，田方四十五步，池环径六步。

术曰：立天元一为环之内圆径，如积求之。得一百二十九万六千五百四十为益实，一万四千七百四十九为从方，四百二十七为从廉，二十五为从隅，立方开之，[3] 得内圆径。合问。



【注释】

[1] 取 $\pi=3$，记环池之径、内圆径、田方分别为 d_1, d_2, a，此即：

$\{a^2-[\frac{3}{4}(2d_1+d_2)^2-\frac{3}{4}d_2^2]\}d_2-[3(2d_1+d_2)]^2=25164$。（郭）

[2] 取 $\sqrt{2}=\frac{7}{5}$，此即：$\frac{1}{2}[\frac{7}{5}a-(2d_1+d_2)]=11\frac{1}{2}$。$3(2d_1+d_2)-3d_2=36$。（郭）

[3] 开方式的现代形式为：$25x^3+427x^2+14749x-1296540=0$。（陈）

be l_1 and l_2. From the statement we have $\frac{1}{12}l_1^2 - \frac{1}{12}l_2^2 = 612$. (G)

[2] Let shi jing be d. From the statement we have $l_2 - d^2 = 48$, $d^2 = l_1 - l_2$. (G)

[3] The expression in modern form is the equation: $x^4 + 32x^2 - 2448 = 0$. (C)

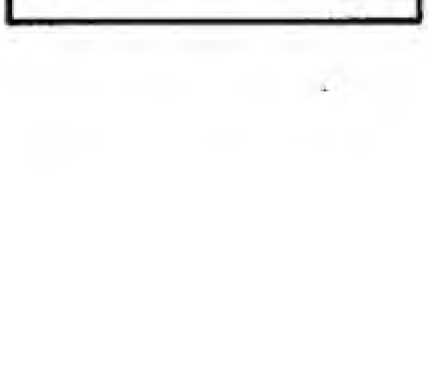
2. A stream of water in the form of a ring is found in a square tract of land. Multiply the area of the irregular part of the land, that without the stream, by the diameter of the circular part, that within the stream, and subtract from it the square of the outer circumference. The remainder is 2564 *bu*.[1] It is said that the distances from the corners of the land to the edge of the water are equal to $11\frac{1}{2}$ *bu*; and the difference between the circumferences is 36 *bu*.[2] Find the three matters.

Ans. The diameter of the land surrounded by the water, 28 *bu*;

a side of the square, 45 *bu*;

width of the stream, 6 *bu*.

Process. Let the element *tian* be the diameter of the inner circle. From the statement we have 1296540 for the negative *shi*, 14749 for the positive *fang*, 427 for the positive *lian*, and 25 for the positive *yu*, a cubic expression[3] whose root is the required diameter.



【今译】

今有一段方田，一环池占据中央，剩余的面积以环池的内圆径乘之，减去外圆周的幂，剩余25164步。只云：从方田的四角到环池的距离各长$11\frac{1}{2}$步，环池的内、外周之差为36步。问：环池内圆径、方田的边长、环池的实径三者各为多少？

答：内圆径28步，方田的边长45步，环池的实径6步。

术：设天元一为环池的内圆径，以如积方法求其解。得到-1296540为常数项，14749为一次项系数，427为二次项系数，25为最高次项系数，开立方，得到内圆径。符合所问。

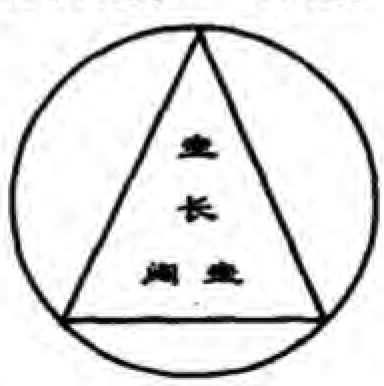
3.

【原文】

今有圆田一段，内有圭池[1]容边占之。只云圭长[2]不及圆径三步半，却多池阔[3]十步半。[4]问：池长、阔及圆径各几何？

答曰：池阔二十一步，池长三十一步半，圆径三十五步。

术曰：立天元一为池阔，如积求之。得一百四十七为正实，一十四为从方，一为益隅，平方开之，[5]得池阔。合问。



【注释】

[1] 等腰三角形的水池。（陈）

[2] 三角形的高。（陈）

[3] 三角形的底。（陈）

[Notes]

[1] Make $\pi = 3$, and let the diameter of the stream of water in the form of a ring be d_1, the diameter of the land surrounded by the water d_2, and a side of the square a. From the statement we have $\{a^2 - [\frac{3}{4}(2d_1 + d_2)^2 - \frac{3}{4}d_2^2]\} d_2 - [3(2d_1 + d_2)]^2 = 25164$. (G)

[2] Let $\sqrt{2} = \frac{7}{5}$, that is, $\frac{1}{2}[\frac{7}{5}a - (2d_1 + d_2)] = 11\frac{1}{2}$, $3(2d_1 + d_2) - 3d_2 = 36$. (G)

[3] The expression in modern form is the equation: $25x^3 + 427x^2 + 14749x - 1296540 = 0$. (C)

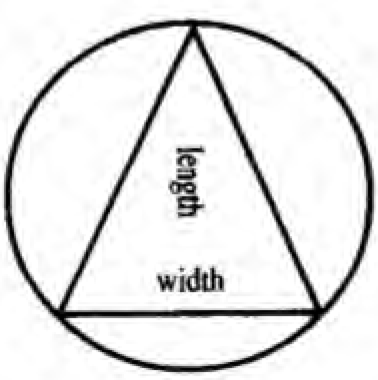
3. A *gui chi* [1] is inscribed in a circular piece of land. It is said that the length[2] of the *gui chi* is less than the diameter of the circle by $3\frac{1}{2}$ *bu*, but exceeds the width[3] of the pond by $10\frac{1}{2}$ *bu*.[4] Find the length and the width of the pond ad the diameter of the circle.

Ans. Width of the pond, 21 *bu*;

length of the pond, $31\frac{1}{2}$ *bu*;

diameter of the circle, 35 *bu*.

Process. Let the element *tian* be the width of the pond. From the statement we have 147 for the positive *shi*, 14 for the positive *fang*, and 1 for the negative *yu*, a quadratic expression[5] whose root is the required width.



［4］记池阔、长、圆径分别为a,b,d，此即：$d-b=3\frac{1}{2}$，$b-a=10\frac{1}{2}$。（郭）

［5］开方式的现代形式为：$-x^2+14x+147=0$。（陈）

【今译】

今有一圆田，内接一等腰三角形水池。只云：此水池的长（三角形的高）比圆径少$3\frac{1}{2}$步，却比水池的阔（三角形的底）多$10\frac{1}{2}$步。问：水池的长、阔及圆径各为多少？

答：水池的阔21步，水池的长$31\frac{1}{2}$步，圆径35步。

术：设天元一为水池的阔，以如积方法求其解。得到147为常数项，14为一次项系数，−1为最高次项系数，开平方，得到水池的阔。符合所问。

4.

【原文】

今有方田一段，靠东北角[1]有圆池占之，余积一万二百二十五步。[2]只云从田西南隅斜至池楞五十九步。[3]问：田方、池径各几何？

答曰：池径一百二十步，田方一百四十五步。

术曰：立天元一为池径，如积求之。得五十五万二千为益实，四千七百二十为从方，一为益隅，平方开之，[4]得池径。合问。

[Notes]

[1] A pond of water in the form of an isosceles triangle. (C)

[2] The altitude of the triangle. (C)

[3] The base of the triangle. (C)

[4] Let the width and the length of the pond be a and b, and the diameter of the circle d. From the statement we have $d - b = 3\frac{1}{2}, b - a = 10\frac{1}{2}$. (G)

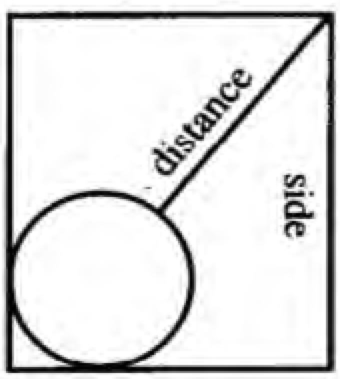
[5] The expression in modern form is the equation: $-x^2 + 14x + 147 = 0$. (C)

4. There is a circular pond of water at the northeast [1] corner of a square piece of land; the areas of the land itself is 10225 *bu*.[2] It is said that the distance from the southwest corner to the edge of the water is 59 *bu*.[3] Find a side of the square and the diameter of the pond.

Ans. Diameter of the pond, 120 *bu*;

side of the square, 145 *bu*.

Process. Let the element *tian* be the diameter of the pond. From the statement we have 552000 for the negative *shi*, 4720 for the positive *fang*, and



【注释】

[1]此处所使用的方向是根据中国古法。如同许多其他事情一样恰好与西方方向相反。因此，此图顶部是南，右边为西，左边为东。(陈)

[2]取$\pi=3$，记方田的一边、池径分别为a,d，此即：$a^2-\frac{3}{4}d^2=10225$。(郭)

[3]取$\sqrt{2}=\frac{7}{5}$，此即：$\frac{7}{5}\times\frac{d}{2}+\frac{d}{2}+59=\frac{7}{5}a$。(郭)

[4]开方式的现代形式为：$-x^2+4720x-552000=0$。(陈)

【今译】

今有一方田，靠东北角有一圆形水池占之，剩余的面积为10225步。只云：从方田的西南角斜至圆池的距离是59步。问：方田的边长、圆池的直径各为多少?

答：圆池的直径120步，方田的边长145步。

术：设天元一为圆池的直径，以如积方法求其解。得到-552000为常数项，4720为一次项系数，-1为最高次项系数，开平方，得到圆池的直径。符合所问。

5.

【原文】

今有圆田一段，周一百二十步，被水从中穿为直河，分为弧田二段。只云二弧弦各长三十二步。[1] 问：水面阔几何?

答曰：二十步。

术曰：立天元一为水面阔，如积求之。得五百七十六为益实，一为从隅，平方开之，[2] 得水面阔。合问。

1 for the negative *yu*, a quadratic expression[4] whose root is the diameter of the pond.

[Notes]

[1] The direction used in this book is according to the old Chinese method which, as in many other things, is directly opposite to the Western method. Thus the top of a page is south, the bottom north, the right-hand side west, and the left-hand side east. (C)

[2] Make $\pi = 3$, and let a side of the square be a, and the diameter of the pond d. From the statement we have $a^2 - \frac{3}{4}d^2 = 10225$. (G)

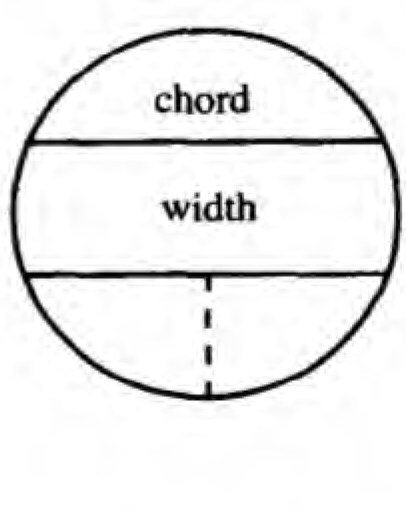
[3] Let $\sqrt{2} = \frac{7}{5}$, that is, $\frac{7}{5} \times \frac{d}{2} + \frac{d}{2} + 59 = \frac{7}{5}a$. (G)

[4] The expression in modern form is the equation: $-x^2 + 4720x - 552000 = 0$. (C)

5. A stream passes directly through a circular piece of land dividing it into two segments. The chord of each segment is equal to 32 *bu* and the circumference of the land is equal to 120 *bu*.[1] Find the width of the stream.

Ans. 24 *bu*.

Process. Let the element *tian* be the width of the stream. From the statement we have 576 for the negative *shi*, and 1 for the positive *yu*, a quadratic expression[2] whose root is the required width.



【注释】

[1] 取 $\pi=3$，记圆周、圆径、弧弦、水面阔分别为 l，d，c，a，则 $d=\frac{l}{3}=40$。由勾股术，$a^2=d^2-c^2=40^2-32^2=576$。（郭）

[2] 开方式的现代形式为：$x^2-576=0$。（陈）

【今译】

今有一圆田，周长为120步，被水从中间穿过，成为一条直河，将圆田分成二段弧田。只云：二弧田的弦为32步。问：水面阔为多少？

答：水面阔24步。

术：设天元一为水面阔，以如积方法求其解。得到-576为常数项，1为最高次项系数，开平方，得到水面阔。符合所问。

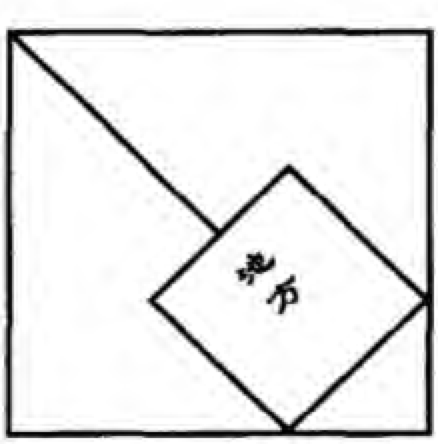
6.

【原文】

今有方田一段，靠西北隅有结角方池占之，余积四千步。[1] 只云从田东南隅斜至池楞六十八步八分。[2] 问：田、池各方几何？

答曰：池方十五步，田方六十五步。

术曰：立天元一为池方，如积求之。得七万七千六百六十四为益实，五千九十一步二分为从方，五步七分六厘为从隅，平方开之，[3] 得池方。合问。



[Notes]

[1] Make $\pi = 3$, and let the circumference be l, the diameter d, the chord of each segment c, and the width of the stream a . Then, $d = \frac{l}{3} = 40$. From the *gou gu* method, $a^2 = d^2 - c^2 = 40^2 - 32^2 = 576$. (G)

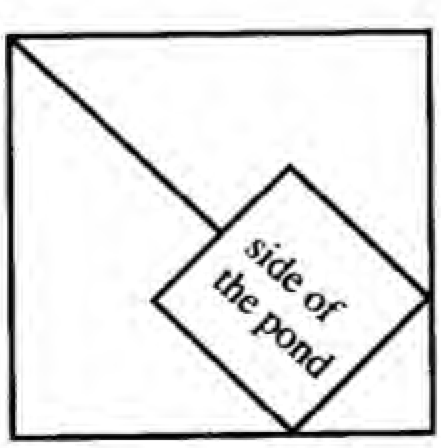
[2] The expression in modern form is the equation: $x^2 - 576 = 0$. (C)

6.A square pond of water is situated obliquely in the northwest corner of a square piece of land. The area of the land is 4000 *bu*.[1] It is said that the distance from the southeast corner to the edge of the water is 68 *bu* 8 *fen*.[2] Find the sides of the land and the pond.

Ans. Side of pond, 15 *bu*;

side of land, 65 *bu*.

Process. Let the element *tian* be a side of the pond. From the statement we have 77664 for the negative *shi*, 5091 *bu* 2 *fen* for the positive *fang*, and 5 *bu* 7 *fen* 6 *li* for the positive *yu*, a quadratic expression[3] whose root is the required side.



【注释】

［1］记方田、方池的一边分别为a_1,a_2，此即：$a_1^2-a_2^2=4000$。（郭）

［2］"步"下之"八分"即0.8步。取$\sqrt{2}=\frac{7}{5}$，此即：$\frac{7}{5}a_1-(a_2+\frac{1}{2}a_2)=68.8$。（郭）

［3］"步"下之"二分"、"七分六厘"分别即0.2步、0.76步。（郭）开方式的现代形式为：$5.76x^2+5091.2x-77664=0$。（陈）

【今译】

今有一方田，靠西北角有一与两边结角的方形水池占之，剩余的面积为4000步。只云：从方田的东南角斜至方池的距离是68.8步。问：方田、方池的边长各为多少？

答：方池的一边15步，方田的一边65步。

术：设天元一为方池的一边长，以如积方法求其解。得到−77664为常数项，5091.2为一次项系数，5.76为最高次项系数，开平方，便得到方池的一边长。符合所问。

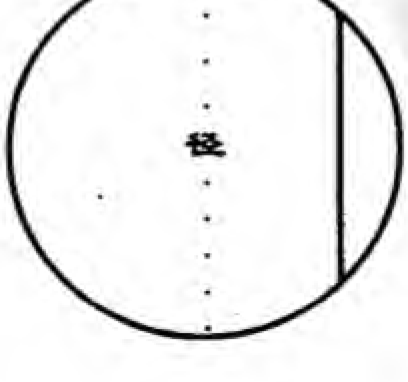
7.

【原文】

今有圆田一段，西边被水侵入一弧，外有残周五十三步，弦长二十步。问：圆径、弧背、矢阔各几何？

答曰：圆径二十五步，矢阔五步，弧背二十二步。

术曰：立天元一为水侵弧矢，如积求之。得三万为正实，七千三百为益方，六百为从上廉，七十三为益下廉，一为正隅，三乘方开之，[1]得矢阔。　又：矢除半弦幂，加矢，即圆田径。[2]　又：倍矢幂，以圆径除之，为弦背差。加弦，即弧背。[3]合问。



[Notes]

[1] Let the side of the land be a_1, and the side of the pond a_2. From the statement we have $a_1^2 - a_2^2 = 4000$. (G)

[2] 8 *fen* following the *bu* is 0.8 *bu*. Make $\sqrt{2} = \frac{7}{5}$, that is, $\frac{7}{5}a_1 - (a_2 + \frac{1}{2}a_2) = 68.8$. (G)

[3] 2 *fen* and 7 *fen* 6 *li* following *bu* are 0.2 *bu* and 0.76 *bu* respectively. The expression in modern form is the equation: $5.76x^2 + 5091.2x - 77664 = 0$. (C)

7. A segment on the west side of a circular piece of land is occupied by water. The remainder of the circumference is 53 *bu* and its chord 20 *bu*. Find the diameter of the circle, and the arc and the *shi* of the segment.

Ans. Diameter, 25 *bu*;

shi, 5 *bu*;

arc, 22 *bu*.

Process. Let the element *tian* be *shi* of the segment occupied by the water. From the statement we have 30000 for the positive *shi*, 7300 for the negative *fang*, 600 for the positive first *lian*, 73 for the negative last *lian*, and 1 for the positive *yu*, an expression[1] of the fourth degree whose root is the required *shi*. The sum of the square of one-half of the chord divided by the *shi* and the *shi* is equal to the diameter of the circle;[2] and twice the square of the *shi* divided by the diameter of the circle is the difference between the chord and its arc. Increase this difference by the chord, and we then have the required arc of the segment.[3]

【注释】

［1］开方式的现代形式为：$x^4-73x^3+600x^2-7300x+30000=0$。（陈）

［2］取$\pi=3$，记圆周、圆径、弧弦、弧背、矢阔分别为l，d，c，l_1，v，则由《九章算术》勾股锯圆材术，$d=\frac{(\frac{c}{2})^2}{v}+v$。（郭）

［3］由沈括会圆术，$l=c+\frac{2v^2}{d}$。（郭）

【今译】

今有一圆田，西边被水侵入一弧田。外有残周53步，弦长20步。问：圆径、弧背、矢阔各为多少？

答：圆径25步，矢阔5步，弧背22步。

术：设天元一为水侵弧的矢阔，以如积方法求其解。得到30000为常数项，−7300为一次项系数，600为二次项系数，−73为三次项系数，1为最高次项系数，开四次方，得到矢阔。　又：半弦幂除以矢阔，加矢阔，就是圆径。　又：将矢阔之幂加倍，以圆径除之，为弦与弧背之差。加上弦，就是弧背。符合所问。

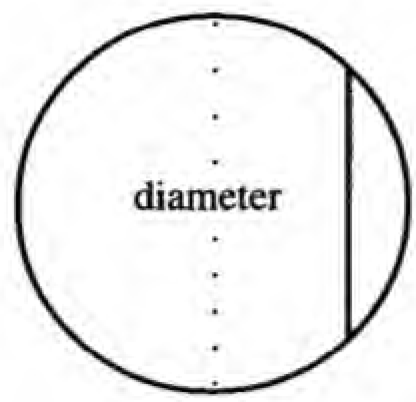
8.

【原文】

今有方田一段，西北隅被水侵占之，余积七千一百一十二步半。[1] 只云东南隅斜至水楞一百八步半。[2] 问：田方及水长各几何？

答曰：田方八十五步，水长二十一步。

术曰：立天元一为田方面，如积求之。得一万九千一百二十五为正实，三百一十为益方，一为正隅，平方开之。[3] 所得，七之，五而一，为田斜。内减云数，余为池斜。倍之，即水长。合问。



[Notes]

[1] The expression in modern form is the equation: $x^4 - 73x^3 + 600x^2 - 7300x + 30000 = 0$. (C)

[2] Make $\pi = 3$, and let the circumference be l, the diameter d, the chord c, the arc l_1, and the *shi* v. According to the *gou gu ju yuan cai* method in *The Nine Chapters of Mathematical Procedures*, $d = \frac{(\frac{c}{2})^2}{v} + v$. (G)

[3] Acording to Shen Kuo's *hui yuan* method, $l = c + \frac{2v^2}{d}$. (G)

8. After taking of the northwest corner, occupied by water, of a square piece of land the remainder is equal to $7112\frac{1}{2}$ (square) *bu*.[1] It is said that the distance from the southeast corner to the edge of the water is $108\frac{1}{2}$ *bu*.[2] Find a side of the land and the length of the edge of the water.

Ans. Side of the land, 85 *bu*;

edge of the water, 21 *bu*.

Process. Let the element *tian* be a side of the square. From the statement we have 19125 for the positive *shi*, 310 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression [3] whose root is a side of the square.

【注释】

［1］取记方田的一边及水侵三角形的底分别为a_1、a_2，此即：$a_1^2-(\frac{1}{2}a_2)^2=7112\frac{1}{2}$。（郭）

［2］取$\sqrt{2}=\frac{7}{5}$，此即：$\frac{7}{5}a_1-\frac{1}{2}a_2=108\frac{1}{2}$。（郭）

［3］开方式的现代形式为：$x^2-310x+19125=0$。（陈）

【今译】

今有一方田，西北角被水侵占，剩余的面积为$7112\frac{1}{2}$步。只云：从方田的东南角斜至水边的距离是$108\frac{1}{2}$步。问：方田的边长与水侵三角形的底各为多少？

答：方田的一边85步，水侵三角形的底21步。

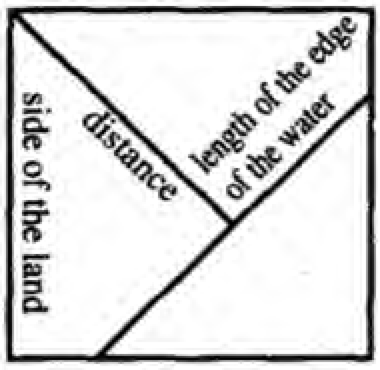
术：设天元一为方田的一边长，以如积方法求其解。得到19125为常数项，−310为一次项系数，1为最高次项系数，开平方，就是方田的一边长。其边长，以7乘之，以5除之，为方田的斜长。内中减去东南角至水边的距离，余数为水的斜长。加倍，就是水侵三角形的底。符合所问。

9.

【原文】

今有方五斜七八角田一段，内复有方五斜七八角池占之，余积三千九百

Seventh-fifths of this root is the diagonal of the land. Subtracting from this diagonal the distance to the edge of the water we then have the diagonal of the pond. Twice this diagonal is the required length of the edge.



[Notes]

[1] Let a side of the land be a_1, and the length of the edge of the water a_2. From the statement we have $a_1^2 - (\frac{1}{2}a_2)^2 = 7112\frac{1}{2}$. (G)

[2] Make $\sqrt{2} = \frac{7}{5}$, that is, $\frac{7}{5}a_1 - \frac{1}{2}a_2 = 108\frac{1}{2}$. (G)

[3] The expression in modern form is the equation: $x^2 - 310x + 19125 = 0$. (C)

9. A piece of land in the form of a *fang-wu-xie-qi* octagon has a pond of water of the similar form in the center. The area of the dry land is $3977\frac{7}{49}$ (square) *bu*.[1] It is said that the *mian jing* to the edge of the water all

七十七步四十九分步之七。[1]只云面径至池楞各长一十七步。[2]问：田、池面各阔几何？

答曰：田阔三十六步，池阔二十二步。

术曰：立天元一为池面阔，如积求之。得三十六万二千二百八为益实，一万六千四百六十四为从方，开无隅平方而一，[3]得池阔。加差一十四，即外田面阔。合问。

【注释】

[1] 取$\sqrt{2}=\frac{7}{5}$，记八角田与八角池的一边分别为a_1,a_2，面积分别为S_1,S_2此即：$S_2=a_2^2+4\times a_2\times\frac{5}{7}a_2+4\times\frac{1}{2}(\frac{5}{7}a_2)^2=\frac{239}{49}a_2^2$，$S_1=\frac{239}{49}a_1^2$，

$S_1-S_2=3977\frac{7}{49}$。（郭）

[2] 此即：$a_1+2\times\frac{5}{7}a_1=a_2+2\times\frac{5}{7}a_2+2\times 17$，或$a_1=a_2+14$。（郭）

[3] 开方式的现代形式为：$16464x-362208=0$。（陈）

【今译】

今有一方5斜7八角田，内中有一方5斜7八角池占之，剩余的面积为$3977\frac{7}{49}$步。只云：从八角田的外边至八角池的距离是17步。问：八角田与八角池的一边长各为多少？

答：八角田的一边36步，八角池的一边长22步。

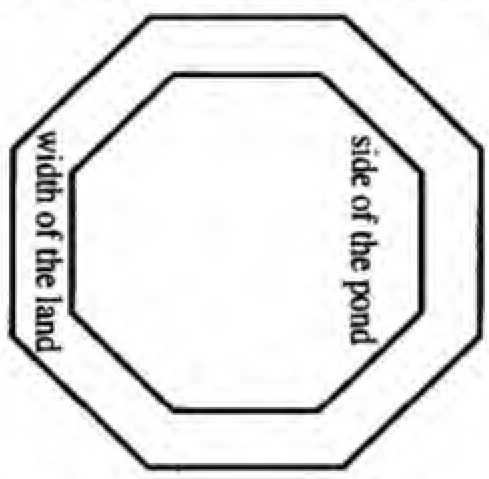
术：设天元一为八角池的一边长，以如积方法求其解。得到-362208为常数项，16464为一次项系数，开无隅平方除之，得到八角池的一边长。加两者之差14，就是外八角田的一边长。符合所问。

equal to 17 *bu*.[2] Find the width of the land and a side of the pond.

Ans. Width of the land, 36 *bu*;

side of the pond, 22 *bu*.

Process. Let the element *tian* be a side of the pond. From the statement we have 362208 for the negative *shi*, and 16464 for the positive fang, a linear expression[3] whose root is a side of the pond. Adding to a side of the pond the difference, 14, we have the required side of the land.



[Notes]

[1] Make $\sqrt{2}=\frac{7}{5}$, and let the width and area of the land be a_1 and S_1, and a side and the area of the pond a_2 and S_1. From the statement we have $S_2 = a_2^2 + 4 \times a_2 \times \frac{5}{7}a_2 + 4 \times \frac{1}{2}(\frac{5}{7}a_2)^2 = \frac{239}{49}a_2^2$, $S_1 = \frac{239}{49}a_1^2$,

$S_1 - S_2 = 3977\frac{7}{49}$. (G)

[2] That is, $a_1 + 2 \times \frac{5}{7}a_1 = a_2 + 2 \times \frac{5}{7}a_2 + 2 \times 17$, or $a_1 = a_2 + 14$. (G)

[3] The expression in modern form is the equation: $16464x - 362208 = 0$. (C)

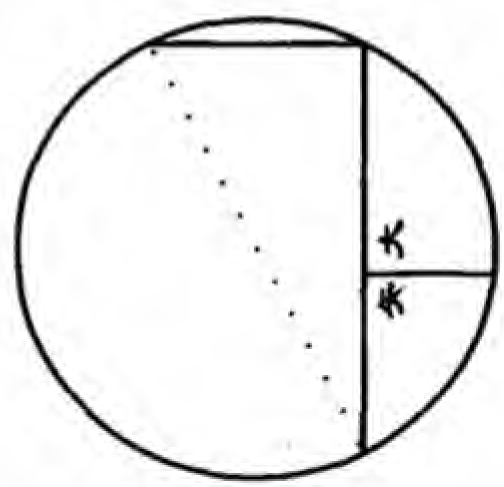
10.

【原文】

今有圆田一段，被水侵入二弧。其大弧弦长二十四步，小弧弦长一十八步。[1] 问：大、小二弧矢各几何？

答曰：大弧矢六步，小弧矢三步。

术曰：立天元一为大弧矢，如积求之。得一百四十四为益实，三十为从方，一为益隅，平方开之，[2] 得大弧矢。 又：立天元一为小弧矢，如积求之。得八十一为正实，三十为益方，一为正隅，平方开之，[3] 得小弧矢。合问。



【注释】

[1] 这里二弧之弦与圆径形成一勾股形，记圆径及二弧的弦、矢分别为 d, c_1, v_1, c_2, v_2，故 $d = \sqrt{c_1^2 + c_2^2} = 30$。由《九章算术》勾股锯圆材术，$d = \frac{(\frac{c}{2})^2}{v} + v$。（郭）

[2] 开方式的现代形式为：$-x^2 + 30x - 144 = 0$。（陈）

[3] 开方式的现代形式为：$x^2 - 30x + 81 = 0$。（陈）

【今译】

今有一圆田，被水侵占了二弧田，其大弧的弦长 24 步，小弧的弦长 18 步。问：大、小二弧的矢各为多少？

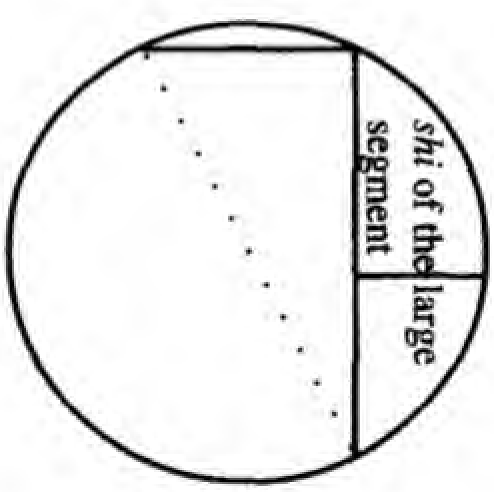
答：大弧矢六步，小弧矢三步。

10. The two segments of a circular piece of land are occupied by water. The chord of the large segment is 24 *bu* and that of the smaller 18 *bu*. [1] Find the shi of both segments.

Ans. Large segment, 6 *bu*;

smaller segment, 3 *bu*.

Process. Let the element *tian* be the *shi* of the large segment. From the statement we have 144 for the negative *shi*, 30 for the positive *fang*, and 1 for the negative *yu*, a quadratic expression [2] whose root is the *shi* of the large segment. Again let the element *tian* be the *shi* of the smaller segment. From the statement we have 81 for the positive *shi*, 30 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[3] whose root is the shi of the smaller segment.



[Notes]

[1] A right triangle is formed by the two segments' chords and the circle's diameter. Let the diameter be d, the chord and the *shi* of one segment c_1 and v_1, and the chord and the *shi* of the other c_2 and v_2. Therefore, $d = \sqrt{c_1^2 + c_2^2} = 30$. According to the *gou gu ju yuan cai* method in *The Nine Chapters of the Mathematical Procedures*, $d = \frac{(\frac{c}{2})^2}{v} + v$. (G)

[2] The expression in modern form is the equation: $-x^2 + 30x - 144 = 0$. (C)

术：设天元一为大弧矢，以如积方法求其解。得到 -144 为常数项，30 为一次项系数，-1 为最高次项系数，开平方，得到大弧矢。 又：设天元一为小弧矢，以如积方法求其解。得到 81 为常数项，-30 为一次项系数，1 为最高次项系数，开平方，得到小弧矢。符合所问。

11.

【原文】

今有圆田一段，圆从古法。上有圆池，圆从密率。中有直池，池边下有方池，各占之，余积一千八百六十八步四分九厘五豪二丝。[1] 只云：七池方面不及一直池长五步四分四厘，却多三直池阔二步二分四厘；方池面、圆池周和得三十步；直池斜与方池幂等。[2] 问：田、池周、径、长、阔各几何？

答曰：圆田径六十四步，圆池周二十二步；

直池长六十一步四分四厘，阔一十七步九分二厘；

圆池径七步，方池面八步。

术曰：立天元一为圆池周，如积求之。得七千七百八十五万五千一百二十六步八分为正实，一千五十五万三千七百三十四步四分为益方，五十三万二千三百一步五分为从上廉，一万一千八百八十为益下廉，九十九为从隅，三乘方开之，[3] 得圆池周。余，依加减求之。合问。

[3] The expression in modern form is the equation: $x^2 - 30x + 81 = 0$. (C)

11. A large circular tract of land — the ancient value of π — has a circular pond — the *mi* value of π — at the top, a straight stream in the middle, and a square pond at the bottom. The area of the dry land is 1868 *bu* 4 *fen* 9 *li* 5 *hao* 2 *si*.[1] It is said that 7 times a side of the square pond is less than the length of the stream by 5 *bu* 4 *fen* 4 *li*, but exceeds 3 times the width of the stream by 2 *bu* 2 *fen* 4 *li*; the sum of a side of the square pond and the circumference of the circular pond is 30 *bu*; and the diagonal of the stream is equal to the square of a side of the square pond.[2] Find the circumferences, diameters, lengths and width of the land, the circular pond, the square pond and the stream.

Ans. Diameter of the land, 64 *bu*;
circumference of the circular pond, 22 *bu*;
length of the stream, 61 *bu* 4 *fen* 4 *li*;
width of the stream, 17 *bu* 9 *fen* 2 *li*;
diameter of the circular pond, 7 *bu*;
side of the square pond, 8 *bu*.

Process. Let the element *tian* be the circumference of the circular pond. From the statement we have 77855126 *bu* 8 *fen* for the positive *shi*, 10553734 *bu* 4 *fen* for the negative *fang*, 532301 *bu* 5 *fen* for the positive first *lian*, 11880 for the negative last *lian*, and 99 for the positive *yu*, an expression[3] of the fourth degree whose root is the required circumference. The other dimensions can be obtained by addition and subtraction.

【注释】

[1]"步"下之"四分九厘五毫二丝"即0.4952步。记圆田周、径，圆池周、径，直池长、阔，方池边长分别为l，d，l_1，d_1，b_1，a_1，a_2，则根据题设，$\frac{3}{4}d^2-\frac{11}{14}d_1^2-a_1b_1-a_2^2=1868.4952$。（郭）

[2]"步"下之"四分四厘"、"二分四厘"分别即0.44步、0.24步。此即：$b_1-7a_2=5.44$，$7a_2-3a_1=2.24$，$a_2+l_1=30$，$\sqrt{a_1^2+b_1^2}=a_2^2$。（郭）

[3]"步"下之"八分"、"四分"、"五分"分别即0.8步、0.4步、0.5步。（郭）开方式的现代形式为：$99x^4-11880x^3+532301.5x^2-10553734.4x+77855126.8=0$。（陈）

【今译】

今有一圆田，圆周率按古法。上面有一圆池，圆周率按密率。中间有一长方形池，长方形池的下面有一正方形池，各占之，余积1868.4952步。只云：正方形池的一边的7倍比长方形池的长少5.44步，却比长方形池的阔的3倍多2.24步，正方形池的一边与圆池的周之和为30步，长方形池的对角线与正方形池的面积相等。问：圆田和各种池的周长、直径、长、阔各为多少？

答：圆田的直径64步，圆池的周长22步，

长方形池的长61.44步，阔17.92步，

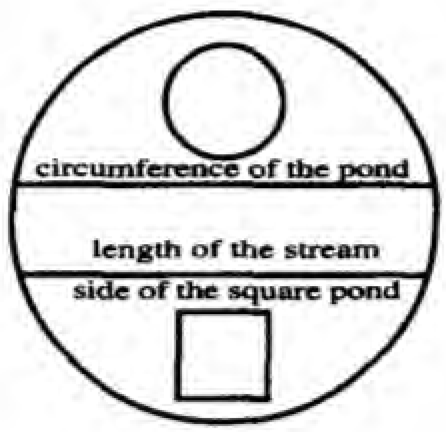
圆池的直径7步，正方形池的一边长8步。

术：设天元一为圆池的周长，以如积方法求其解。得到77855126.8为常数项，−10553734.4为一次项系数，532301.5为二次项系数，−11880为三次项系数，99为最高次项系数，开四次方，得到圆池的周长。其他各项依照加减求之。符合所问。

12.

【原文】

今有方田一段，内有方池，池心复有方亭台，各占之，三积共五千五十



[Notes]

[1] 4 *fen* 9 *li* 5 *hao* 2 *si* following the *bu* is 0.4952 *bu*. Let the circumference and the diameter of the land be l and d, the circumference and the diameter of the circular pond l_1 and d_1, the length and the width of the stream b_1 and a_1, and the side of the square pond a_2. According to the statement we have $\frac{3}{4}d^2 - \frac{11}{14}d_1^2 - a_1b_1 - a_2^2 = 1868.4952$. (G)

[2] 4 *fen* 4 *li* and 2 *fen* 4 *li* following *bu* are 0.44 *bu* and 0.24 *bu* respectively. That is, $b_1 - 7a_2 = 5.44$, $7a_2 - 3a_1 = 2.24$, $a_2 + l_1 = 30$, $\sqrt{a_1^2 + b_1^2} = a_2^2$. (G)

[3] 8 *fen*, 4 *fen* and 5 *fen* following *bu* are 0.8 *bu*, 0.4 *bu*, and 0.5 *bu* respectively. (G) The expression in modern form is the equation: $99x^4 - 11880x^3 + 532301.5x^2 - 10553734.4x + 77855126.8 = 0$. (C)

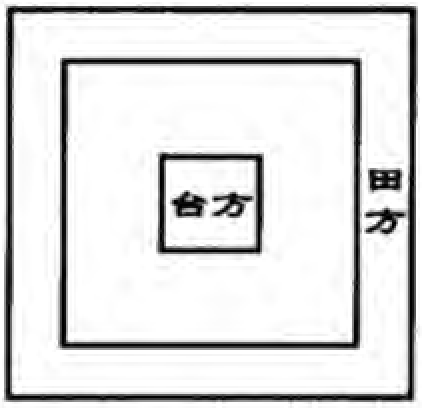
12. A square piece of land has a square pond in the center. In the center of the pond is an observatory with a square tower. The sum of the three *ji* [the area of the dry land, the area of the water, and the volume of the observatory] is

六尺。[1] 只云：并台高、台方为益实，二从方，一益廉，一从隅，立方开之，并入台方面，共得一丈一尺。台高不及池方面九尺。台方面幂与外田方同。[2] 问：三方面及台高各几何？

答曰：田方六十四尺；池方二十五尺；

台高一丈六尺，台方八尺。

术曰：立天元一为开方数，如积求之。得八千一百三十三为正实，四千六百九十七为益方，五百二十七为正上廉，一百二为正二廉，二十二为益下廉，一为正隅，四乘方开之，[3] 得三尺，为开方数。合问。



【注释】

[1] 记方田、方池的边长，以及方台的边长、高分别为 a，a_1，a_2，h，则根据题设，$(a^2-a_1^2)+(a_1^2-a_2^2)+a_2^2h=5056$。（郭）

[2] 此即：若 w 是开方式的 $w^3-w^2+2w-(a_2+h)=0$ 根，则 $w+a_2=11$；$a_1-h=9$，$a=a_2^2$。（郭）

[3] 开方式的现代形式为：$x^5-22x^4+102x^3+527x^2-4697x+8133=0$。（陈）

【今译】

今有一方田，中间有一正方形池，正方形池的中心有一方台，各占之，三积共5056尺。只云：方台的一边加高作为负常数项，2为一次项系数，-1作为二次项系数，1为最高次项系数，开立方。其开方数加方台的边长，得1丈1尺。方台的高比方池的边长少9尺。方台的底面积与方田

5056 *chi*.[1] It is said that taking the sum of the height of the observatory and the length of a side for the negative *shi*, 2 for the positive *fang*, 1 for the negative *lian*, and 1 for the positive *yu* of a cubic expression, its root increased by the length of a side of the observatory is equal to 11 *chi*. The height of the observatory is less than a side of the pond by 9 *chi*. The square of a side of the observatory is equal to a side of the land.[2] Find the sides of the three squares and the height of the observatory.

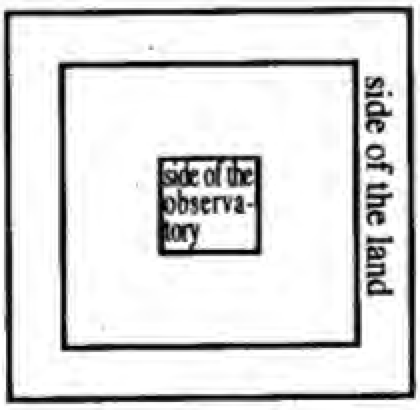
Ans. Side of the land, 64 *chi*;

side of the pond, 25 *chi*;

height of the observatory, 1 *zhang* 6 *chi*;

side of the observatory, 8 *chi*.

Process. Let the element *tian* be a root of the given expression. From the statement we have 8133 for the positive *shi*, 4697 for the negative *fang*, 527 for the positive first *lian*, 102 for the positive second *lian*, 22 for the negative last *lian*, and 1 for the positive *yu*, an expression[3] of the fifth degree whose root, 3, is the required root.



【 Notes 】

[1] Let the side of the land be a, the side of the pond a_1, and the side and the height of the observatory a_2 and h. From the statement we have $(a^2 - a_1^2) + (a_1^2 - a_2^2) + a_2^2 h = 5056$. (G)

的边长相等。问：方田、方池的边长，以及方台的边长、高各为多少？

答：方田的边长64尺，方池的边长22尺，

方台的高1丈6尺，方台的边长8尺。

术：设天元一为上开方式的开方数，以如积方法求其解。得到8133为常数项，-4697为一次项系数，527为二次项系数，102为三次项系数，-22为四次项系数，1为最高次项系数，开五次方，得到3尺，就是开方数。符合所问。

13.

【原文】

今有圆田一段，内有圆池，池中复有圆亭台，各占之，三积共九千五百四尺。[1] 只云：台、池二周皆以平方开之，相并，自之，与外田周等。其台周开方数如池周开方数二分之一，不及台高二尺。[2] 问：三圆周及台高各几何？

答曰：田周三百二十四尺；池周一百四十四尺；

台周三丈六尺，台高八尺。

术曰：立天元一为台高，如积求之。得一十一万二千七百六十八为益实，二千五百四十四为益方，一千八百八十八为从上廉，六百一十六为益二廉，七十二为从下廉，一为正隅，四乘方开之，[3] 得台高。合问。

[2] That is, if w is one of the roots of the equation $w^3 - w^2 + 2w - (a_2 + h) = 0$, then $w + a_2 = 11$; $a_1 - h = 9, a = a_2^2$. (G)

[3] The expression in modern form is the equation: $x^5 - 22x^4 + 102x^3 + 527x^2 - 4697x + 8133 = 0$. (C)

13. A circular field has a circular pond at its center. In the center of the pond is a circular observatory. The sum of the three *ji* [the areas of the land and the pond and the volume of the observatory] is 9504 *chi*.[1] It is said that the square of the sum of the square roots of the circumferences of the observatory and of the pond equals the circumference of the land. The square root of the circumference of the observatory is one-half of the square root of the circumference of the pond and is less than the height of the observatory by 2 *chi*.[2] Find the circumferences of the circles and the height of the observatory.

Ans. Circumference of the land, 324 *chi*;

circumference of the pond, 144 *chi*;

circumference of the observatory, 3 *zhang* 6 *chi*;

height of the observatory, 8 *chi*.

Process. Let the element *tian* be height of the observatory. From the statement we have 112768 for the negative *shi*, 2544 for the negative *fang*, 1888 for the positive first *lian*, 616 for the negative second *lian*, 72 for the positive last *lian*, and 1 for the positive *yu*, an expression [3] of the fifth degree whose root is the height of the observatory.

【注释】

[1] 记圆田、圆池的周长，以及圆台的周长、高分别为l，l_1，l_2，h，则根据题设，$(\frac{1}{12}l^2-\frac{1}{12}l_1^2)+(\frac{1}{12}l_1^2-\frac{1}{12}l_2^2)+\frac{1}{12}l_2^2h=9504$。（郭）

[2] 此即：$(\sqrt{l_1}+\sqrt{l_2})^2=l$，$\sqrt{l_2}=\frac{1}{2}\sqrt{l_1}$，$h-\sqrt{l_2}=2$。（郭）

[3] 开方式的现代形式为：$x^5+72x^4-616x^3+1888x^2-2544x-112768=0$。（陈）

【今译】

今有一圆田，中间有一圆池，圆池的中心有一圆台，各占之，三积共9504尺。只云：圆台、圆池的周长分别开平方，相加，再自乘，与外圆田的周长相等。其圆台周长的开方数是圆池周长的开方数的$\frac{1}{2}$，却比圆台的高少2尺。问：圆田、圆池的周长，以及圆台的周长、高各为多少？

答：圆田的周长324尺，圆池的周长144尺，

圆台的周长3丈6尺，圆台的高8尺。

术：设天元一为圆台的高，以如积方法求其解。得到−112768为常数项，−2544为一次项系数，1888为二次项系数，−616为三次项系数，72为四次项系数，1为最高次项系数，开五次方，得到圆台的高。符合所问。

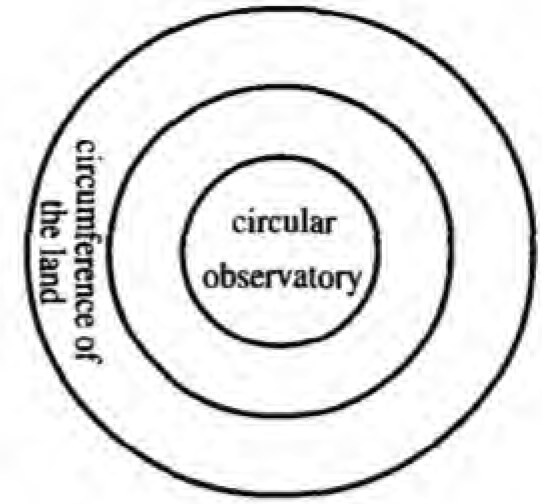
14.

【原文】

今有圭田一段，阔一十四步，长二十四步。于内欲容圆池一所。[1] 问：池径几何？

答曰：一十步二分步之一。

术曰：立天元一为容圆池径，如积求之。得一千一百七十六为益实，



[Notes]

[1] Let the circumference of the land be l, the circumference of the pond l_1, and the circumference and the height of the observatory l_2 and h. From the statement we have $(\frac{1}{12}l^2 - \frac{1}{12}l_1^2) + (\frac{1}{12}l_1^2 - \frac{1}{12}l_2^2) + \frac{1}{12}l_2^2h = 9504$. (G)

[2] That is, $(\sqrt{l_1}+\sqrt{l_2})^2 = l$, $\sqrt{l_2}=\frac{1}{2}\sqrt{l_1}$, $h - \sqrt{l_2} = 2$. (G)

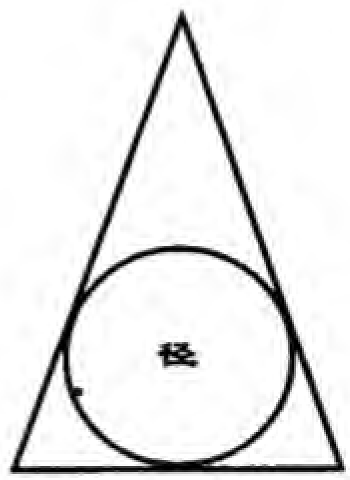
[3] The expression in modern form is the equation: $x^5 + 72x^4 - 616x^3 + 1888x^2 - 2544x - 112768 = 0$. (C)

14. A circular pond is inscribed in a piece of land in the form of an isosceles triangle whose width is 14 *bu* and length 24 *bu*.[1] Find the diameter of the circle.

Ans. $10\frac{1}{2}$ *bu*.

Process. Let the element *tian* be the diameter of the inscribed circle. From

四十九为从方，六为从隅，平方开之，[2] 得圆径。合问。



【注释】

［1］记圭田的阔、长及所容的圆径分别为 a，b，d，由面积得 $ab=\left(\sqrt{\frac{1}{4}a^2+b^2}+\frac{1}{2}a\right)d$。（郭）

［2］开方式的现代形式为：$6x^2+49x-1176=0$。（陈）

【今译】

今有一圭田，阔 14 步，长 24 步。欲于其中容一圆池。问：圆池径为多少？

答：$10\frac{1}{2}$ 步。

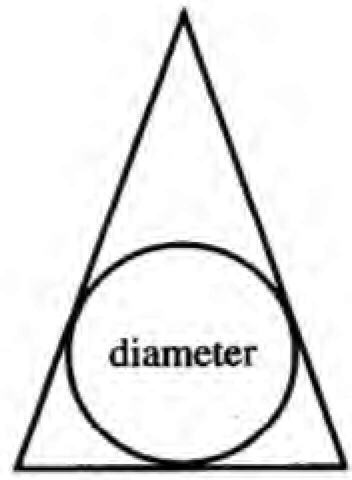
术：设天元一为圆池直径，以如积方法求其解。得到 -1176 为常数项，49 为一次项系数，6 为最高次项系数，开平方，得到圆径。符合所问。

15.

【原文】

今有勾股田一段，勾阔一十八步，股长二十四步。今欲从勾内容圆池一所。[1] 问：容池周几何？

the statement we have 1176 for the negative *shi*, 49 for the positive *fang*, and 6 for the positive *yu*, a quadratic expression [2] whose root is the required diameter.



[Notes]

[1] Let the width and the length of the land be a and b, and the diameter of the circular pond d. From the land's area we have: $ab = \left(\sqrt{\frac{1}{4}a^2 + b^2} + \frac{1}{2}a\right)d$. (G)

[2] The expression in modern form is the equation: $6x^2 + 49x - 1176 = 0$. (C)

15. A piece of land in the form of a right triangle has its *gou* equal to 18 *bu* and its *gu* 24 *bu*. Inscribe a circular pond in the piece of land.[1] What will be its circumference?

Ans. 36 *bu*.

答曰：三十六步。

术曰：立天元一为容池周，如积求之。得七千七百七十六为正实，二百五十二为益方，一为正隅，平方开之，[2] 得池周。合问。

【注释】

[1] 记勾股田的勾、股、容圆径分别为 a，b，d，由《九章算术》勾股容圆术：$d=\frac{2ab}{a+b+\sqrt{a^2+b^2}}$。（郭）

[2] 开方式的现代形式为：$x^2-252x+7776=0$。（陈）

【今译】

今有一勾股田，勾 18 步，股 24 步。今欲于勾上容一圆池。问：圆池的周长为多少？

答：36 步。

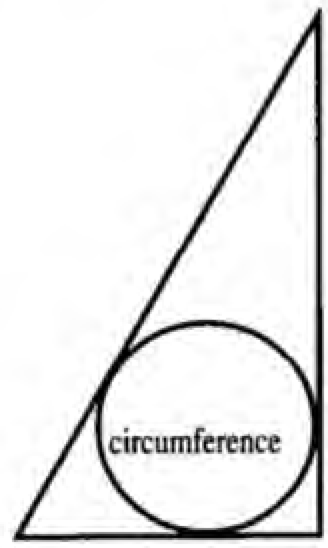
术：设天元一为圆池的周长，以如积方法求其解。得到 7776 为常数项，-252 为一次项系数，1 为最高次项系数，开平方，即得符合所问。

16.

【原文】

今有勾股田一段，勾阔六步，股长一十二步。今欲从勾容方池一所。[1] 问：容方面几何？

Process. Let the element *tian* be the circumference of the inscribed circle. From the statement we have 7776 for the positive *shi*, 252 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[2] whose root is the required circumference.



【 Notes 】

[1] Let the *gou* and the *gu* of the land be a and b, and the diameter of the inscribed pond d. According to the *gou gu rong yuan* method in *The Nine Chapters of Mathematical Procedures*, $d = \frac{2ab}{a + b + \sqrt{a^2 + b^2}}$. (G)

[2] The expression in modern form is the equation: $x^2 - 252x + 7776 = 0$. (C)

16. The *gou* of a right triangular field is 6 *bu* and the *gu* 12 *bu*. If a square pond is dug respecting the *gou*,[1] what is the value of a side of the square?

Ans. 4 *bu*.

Process. Let the element *tian* be a side of the square respecting the *gou*.

答曰：四步。

术曰：立天元一为容方面，如积求之。得七十二为益实，一十八为从方，开无隅平方而一，[2] 得容方面。合问。

【注释】

[1] 记勾股田的勾、股、所容方池的边长分别为 a, b, d, 由《九章算术》勾股容方术：$d=\frac{ab}{a+b}$。(郭)

[2] 开方式的现代形式为：$18x-72=0$。(陈)

【今译】

今有一勾股田，勾6步，股12步。今欲从勾上容一方池。问：所容方池的边长为多少？

答：4步。

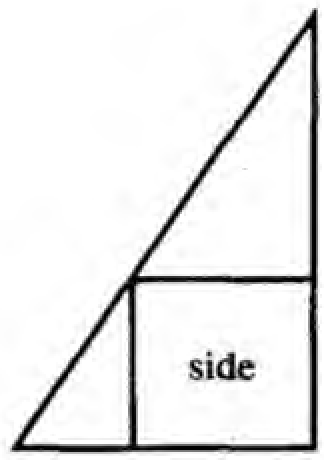
术：设天元一为所容方池的边长，以如积方法求其解。得到 -72 为常数项，18为一次项系数，开无隅平方，得所容方池的边长。符合所问。

17.

【原文】

今有梯田[1] 一段，小阔八步，大阔三十二步，长二十二步半。欲于大阔容圆池一所。[2] 问：容池径几何？

From the statement we have 72 for the negative *shi* and 18 for the positive *fang* of a linear expression.[2] Solving this expression by division we obtain the required side.



[Notes]

[1] Let the *gou* and the *gu* of the right triangular field be a and b, and the side of the square pond d. According to the *gou gu rong fang* method in *The Nine Chapters of Mathematical Procedures*, $d = \frac{ab}{a+b}$. (G)

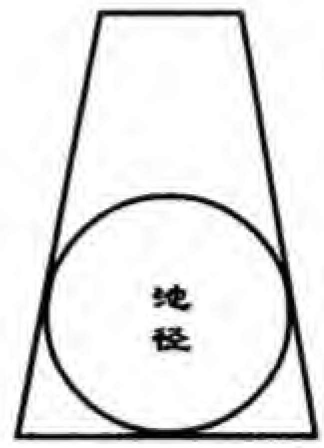
[2] The expression in modern form is the equation: $18x - 72 = 0$. (C)

17. A ladder-formed[1] field has its small width 8 *bu*, large width 32 *bu*, and length $22\frac{1}{2}$ *bu*. If a circular pond is dug respecting the large width, [2] what will be its diameter?

Ans. 19 *bu* 2 *fen*.

答曰：一十九步二分。

术曰：立天元一为大阔容圆径，如积求之。得一十三万八千二百四十为益实，四千六百八为从方，一百三十五为从隅，平方开之，[3]得容圆径。合问。



【注释】

[1] 此为等腰梯形。小阔为其上底，大阔为其下底。（陈）

[2] 记梯田的小阔、大阔、长，所容圆径分别为a_1，a_2，b，d，又记圆池上平行于小、大阔的中阔为a_3，则$\frac{b-d}{b}=\frac{a_3-a_1}{a_2-a_1}$，$\frac{d}{b}=\frac{a_2-a_3}{a_2-a_1}$。（郭）

[3] 开方式的现代形式为：$135x^2+4608x-138240=0$。（陈）

【今译】

今有一梯田，小阔8步，大阔32步，长$22\frac{1}{2}$步。今欲于大阔上容一圆池。问：所容圆池径为多少？

答：19.2步。

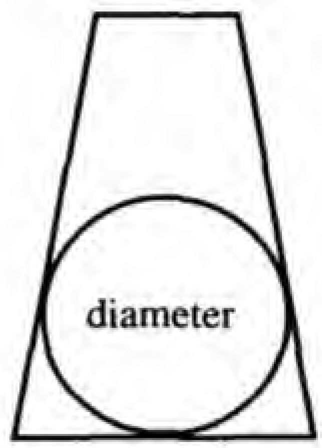
术：设天元一为圆池周，以如积方法求其解。得到-138240为常数项，4608为一次项系数，135为最高次项系数，开平方，得所容圆池的直径。符合所问。

18.

【原文】

今有梯田一段，大阔三十二步，小阔八步，长二十二步半。欲于小头容

Process. Let the element *tian* be the diameter of the pond. From the statement we have 138240 for the negative *shi*, 4608 for the positive *fang*, and 135 for the positive *yu*, a quadratic expression [3] whose root is the required diameter.



[Notes]

[1] A figure in the ladder-form is an isosceles trapezoid. Its small width is the upper base and its large width the lower base. (C)

[2] Let the ladder-formed field's small width be a_1, large width a_2, length b, and the circular pond's diameter d. And let the middle width be a_3, which is in the circular pond and is parallel with the small width and the large width. Then, $\frac{b-d}{b} = \frac{a_3 - a_1}{a_2 - a_1}$, $\frac{d}{b} = \frac{a_2 - a_3}{a_2 - a_1}$. (G)

[3] The expression in modern form is the equation: $135x^2 + 4608x - 138240 = 0$. (C)

18. A ladder-formed field has its large width 32 *bu*, small width 8 *bu*, and length $22\frac{1}{2}$ *bu*. If a circular pond is dug respecting the smaller end of the field, [1] what will be its circumference?

圆池一所。[1] 问：容池周几何？

答曰：四十步。

术曰：立天元一为小头容圆径，如积求之。得九百六十为益实，一百二十八为益方，一十五为从隅，平方开之。[2] 不尽，按之分法求之。[3] 合问。

【注释】

[1] 记梯田的小阔、大阔、长，所容圆径分别为 a_1，a_2，b，d，又记圆池下平行于小、大阔的中阔为 a_3，则 $\frac{d}{b}=\frac{a_3-a_1}{a_2-a_1}$，$\frac{b-d}{b}=\frac{a_2-a_3}{a_2-a_1}$。（郭）

[2] 开方式的现代形式为：$15x^2-128x-960=0$。（陈）

[3] 罗士琳于“求之”下补“得容圆径，三之。即池周”九字，无必要。（郭）

【今译】

今有一梯田，大阔 32 步，小阔 8 步，长 $22\frac{1}{2}$ 步。今欲于小头上容一圆池。问：所容圆池周为多少？

答：40 步。

术：设天元一为小头容圆径，以如积方法求其解。得到 −960 为常数项，−128 为一次项系数，15 为最高次项系数，开平方。不尽，按照之分法求之，得所容圆池的周长。符合所问。

Ans. 40 *bu*.

Process. Let the element *tian* be the diameter of the pond. From the statement we have 960 for the negative *shi*, 128 for the negative *fang*, and 15 for the positive *yu*, a quadratic expression.[2] Solving the expression by the *zhi fen* method, since the root is not integral, we have the diameter of the circle.[3]

[Notes]

[1] Let the ladder-formed field's small width be a_1, large width a_2, length b, and the circular pond's diameter d. And let the middle width be a_3, which is in the circular pond and is parallel with the small width and the large width. Then, $\frac{d}{b} = \frac{a_3 - a_1}{a_2 - a_1}$, $\frac{b - d}{b} = \frac{a_2 - a_3}{a_2 - a_1}$. (G)

[2] The expression in modern form is the equation: $15x^2 - 128x - 960 = 0$. (C)

[3] It is unnecessary that Luo Shilin added nine characters "*de rong yuan jing, san zhi, ji chi zhou*" following "*qiu zhi*". (G)

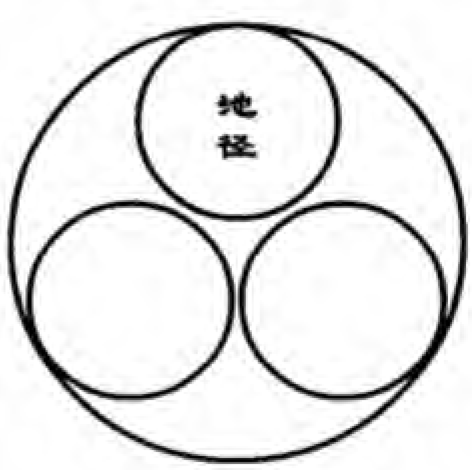
19.

【原文】

今有圆田一段，内有匝边容等径圆池三所。只云：田周减六步，余为益实，一十四为从方，五为益廉，一为正隅，立方开之。得数加入圆径，共得四十八步。[1] 问：三池积几何？

答曰：八百五十五步九十七分步之三十六。

术曰：立天元一为开方数，如积求之。得一百三十八为益实，一十七为从方，五为益廉，一为正隅，立方开之，[2] 得六步。以减云数，余为圆田径。 又：立天元一为容圆池径，如积求之。得五千二百九十二为益实，二百五十二为从方，一为正隅，平方开之，[3] 得池径。不尽，命分。 求池积术曰：列池径，通分内子，自之，于上。分母、分子相减，余，以子乘之。加上，三之，四而一。所得为实。以分母自之，为法。实如法而一。不尽，约之，命分。三之，即三池积。合问。[4]



【注释】

[1] 此取 $\pi = 3$，记圆田周、径分别为 l，d，w 是开方式 $w^3 - 5w^2 + 14w - (l - 6) = 0$ 的根，$w + d = 48$。（郭）

[2] 开方式的现代形式为：$x^3 - 5x^2 + 17x - 138 = 0$。（陈）

[3] 开方式的现代形式为：$x^2 + 252x - 5292 = 0$。（陈）

[4] 该方法的现代表述形式如下：

19. There are three equal circular ponds tangent to each other and tangent internally to a large circular field. It is said if we take the difference between the circumference of the field and 6 for the negative *shi*, 14 for the positive *fang*, 5 for the negative *lian*, and 1 for the positive *yu* of a cubic expression, and its root increased by the diameter is equal to 48 *bu*.[1] Find the sum of the areas of the 3 circular ponds.

Ans. $855\frac{36}{97}$ *bu*.

Process. Let the element *tian* be the root of the given expression. From the statement we have 138 for the negative *shi*, 17 for the positive *fang*, 5 for the negative *lian*, and 1 for the positive *yu*, a cubic expression [2] whose root is 6 *bu*. Subtracting this root from the given number, the remainder is the diameter of the circumscribed circle. Again let the element *tian* be the diameter of one of the equal circles. From the statement we have 5292 for the negative *shi*, 252 for the positive *fang*, and 1 for the positive *yu*, a quadratic expression[3] whose root, a mixed number, is the required diameter. To find the areas of the equal circles, reduce the root found to an improper fraction. Square the numerator, subtract the numerator from the denominator, multiply the remainder by the numerator and add this result to the result obtained above. Take three-fourths of this sum for the dividend, square the denominator for the divisor, and divide. The quotient reduced to lowest terms is the area of one of the smaller circles. This area multiplied by 3 gives the required area of the three circles.[4]

$-(l-6)+14x-5x^2+x^3=0$，这里 l 为大圆的圆周。

则，$l=6+14x-5x^2+x^3$。

但，$l=3d+3(48-x)$，此处 d 为大圆的直径。

$\therefore -128+17x-5x^2+x^3=0$，$x=6$，$48-x=42$，

又令 x 为三个小圆之一的直径，于是我们得到 $3(42-x)=4x^2$，或者

$-5292+252x+x^2=0$。

$$
\begin{array}{llll}
\text{解，} & 1+ & 252- & 5292\ (19\frac{143}{291} \\
 & & \underline{19}+ & \underline{5149} \\
 & & 271- & 143 \\
 & & 19 &
\end{array}
$$

$\therefore x=19\frac{143}{291}$ 或者 $\frac{5672}{291}$。

$[143(291-143)+(5672)^2]\frac{3}{4}\div(291)^2=285\frac{10476}{84681}$，或者 $285\frac{12}{97}$。

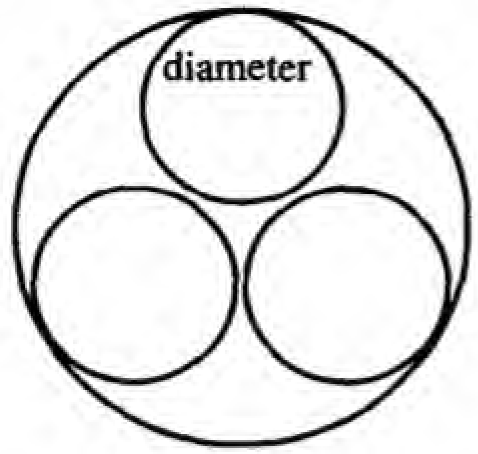
（陈）

【今译】

今有一圆田，其中内切3个两两相切的直径相等的圆。只云：圆田的周减去6步，余数作为负常数项，14为一次项系数，−5为二次项系数，1为最高次项系数，开立方。其得数加入圆田径，共得48步。问：3个圆池的面积共为多少？

答：$855\frac{36}{97}$ 步。

术：设天元一为上开方式的开方数，以如积方法求其解。得到 −138 为常数项，17为一次项系数，−5为二次项系数，1为最高次项系数，开立方，得到6步。以它减48步，就是圆田径。 又：设天元一为所容圆池的直径，以如积方法求其解。得到 −5292 为常数项，252为一次项系数，1为最高次项系数，开平方，得到圆池的直径。不尽，就命名一个分数。 求圆池的面积术：列出圆池的直径，通分内子，自乘，置于上。分母、分子相减，余数以分子乘之，加上者。再以3乘之，除以4，所得作为实。以分母自乘作为法。实除以法，不尽，约之，命名一个分数。乘以3，就是3个圆池的面积。符合所问。



[Notes]

[1] Make $\pi = 3$, and let the circumference and the diameter of the circular pond be l and d, w is one root of the equation $w^3 - 5w^2 + 14w - (l - 6) = 0$, $w + d = 48$. (G)

[2] The expression in modern form is the equation: $x^3 - 5x^2 + 17x - 138 = 0$. (C)

[3] The expression in modern form is the equation: $x^2 + 252x - 5292 = 0$. (C)

[4] The solution in modern form is as follows:

$-(l - 6) + 14x - 5x^2 + x^3 = 0$, where l stands for the circumference of the large circle.

Then $l = 6 + 14x - 5x^2 + x^3$.

But $l = 3d + 3(48 - x)$, where d stands for the diameter of the large circle.

$\therefore\ -128 + 17x - 5x^2 + x^3 = 0$, $x = 6$, $48 - x = 42$, diameter of the circle.

Again let x be the diameter of one of the small circles.

Then we have $3(42 - x) = 4x^2$, or $-5292 + 252x + x^2 = 0$

$$\begin{array}{llll} \text{Solving,} & 1 + 252 & - 5292 & (19\frac{143}{291} \\ & \underline{\quad 19} & + \underline{5149} & \\ & 271 & - \ \ 143 & \\ & \ \ 19 & & \end{array}$$

$\therefore\ x = 19\frac{143}{291}$ or $\frac{5672}{291}$.

$[143(291 - 143) + (5672)^2]\frac{3}{4} \div (291)^2 = 285\frac{10476}{84681}$ or $285\frac{12}{97}$. (C)

方程正负　八问

1.

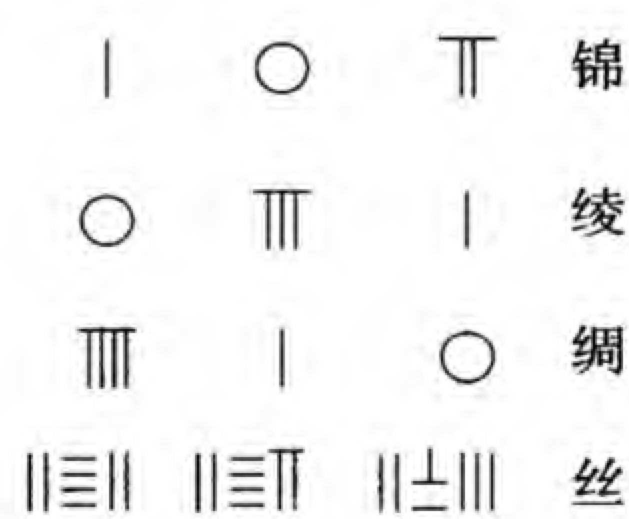
【原文】

今有丝二百七十三两，织锦七匹，织绫一匹；又丝二百四十七两，织绫八匹，织绸一匹；又丝二百四十二两，织绸九匹，织锦一匹。[1] 其锦匹长自乘，内减绫匹长，余又自乘，内加绸匹长，共得三十五万八千八百二十九尺。绫匹长不及绸匹长二尺，却多锦匹长一尺。[2] 问：三色用丝及匹法各长几何？

答曰：锦二丈五尺，丝三十五两；

绫二丈六尺，丝二十八两；

绸二丈八尺，丝二十三两。



术曰：如方程，正负术入之，得三色每匹用丝之数。[3]　立天元一为锦匹长，如积求之。得三十五万八千八百二十五为益实，三为从方，一为益上廉，二为益下廉，一为正隅，三乘方开之，[4]得锦匹长。又：立天元一为绫匹长，如积求之。得三十五万八千八百二十六为益实，五为益方，一十一为从上廉，六为益下廉，一为正隅，三乘方开之，得绫匹长。[5]　又：立天元一为绸匹长，如积求之。得三十五万八千

Fang Cheng Zheng Fu (Simultaneous Equations Positive and Negative)

8 Problems

1. With 273 *liang* of silk one can make 7 pieces of brocade and 1 of damask and with 247 *liang* 8 pieces of damask and 1 of satin; with 242 *liang* 9 pieces of satin and 1 of brocade. [1] Subtracting from the square of the length of the brocade the length of the damask, squaring the remainder and adding the length of the satin, we have 358829 *chi*. The length of the damask is less than the length of the satin by 2 *chi* but exceeds the length of the brocade by 1 *chi*.[2] Find the length of each piece and the amount of silk used each.

Ans. 35 *liang* of silk used in 25 *chi* of brocade;

28 *liang* of silk used in 26 *chi* of damask;

23 *liang* of silk used in 28 *chi* of satin.

Brocade	1	0	7
Damask	0	8	1
Satin	9	1	0
Silk	242	247	273

Process. Find the amount of silk needed in each of the three colors by the law for solving simultaneous equations with positive and negative terms. [3] Let the element *tian* be the length of the brocade. From the statement we have 358825 for the negative *shi*, 3 for the positive *fang*, 1 for the negative upper *lian*, 2 for the negative lower *lian*, and 1 for the positive *yu*, an expression[4]

七百八为益实，一百五十三为益方，七十一为从上廉，一十四为益下廉，一为正隅，三乘方开之，[6] 得绸匹长。合问。

【注释】

[1] 记锦、绫、绸每匹用丝数分别为 x, y, z，则题目的筹式相当于线性方程组，

$$\begin{cases} 7x + y + 0 = 273 \\ 0 + 8y + z = 247 \text{。（郭）} \\ x + 0 + 9z = 242 \end{cases}$$

[2] 记锦、绫、绸的匹长分别为 a，b，c。此即：$(a^2 - b)^2 + c = 358829$，$c - b = 2$，$b - a = 1$。（郭）

[3]	7锦	1绫	0绸	273丝	（1）
	0锦	8绫	1绸	247丝	（2）
	1锦	0绫	69绸	1842丝	（3）
（3）×7，	7锦	0绫	−63绸	1694丝	（4）
（1）−（4），	0锦	1绫	−63绸	−1421丝	（5）
（5）×8，		8绫	−504绸	−11368丝	（6）
（2）−（6），			505绸	11615丝	
			绸，	23两丝。	（陈）

[4] 开方式的现代形式为：$x^4 - 2x^3 - x^2 + 3x - 358825 = 0$。（陈）

[5] 开方式的现代形式为：$y^4 - 6y^3 + 11y^2 - 5y - 358826 = 0$。（陈）

[6] 开方式的现代形式为：$z^4 - 14z^3 + 71z^2 - 153z - 358708 = 0$。（陈）

【今译】

今有丝273两，织锦7匹，织绫1匹。又丝247两，织绫8匹，织绸1匹。又丝242两，织绸9匹，织锦1匹。其锦匹长自乘，减去绫匹长，其余

of the fourth degree whose root is the length of the brocade.

Again let the element *tian* be the length of the damask. From the statement we have 358826 for the negative *shi*, 5 for the negative *fang*, 11 for the positive upper *lian*, 6 for the negative lower *lian*, and 1 for the positive *yu*, an expression [5] of the fourth degree whose root is the length of the damask. Again let the element *tian* be the length of the satin. From the statement we have 358708 for the negative *shi*, 153 for the negative *fang*, 71 for the positive upper *lian*, 14 for the negative lower *lian*, and 1 for the positive *yu*, an expression[6] of the fourth degree whose root is the length of the satin.

【Notes】

[1] Let the amount of silk used by brocade be x, used by damask y, and used by satin z. The equations are the same as the group of linear equations:

$$\begin{cases} 7x + y + 0 = 273 \\ 0 + 8y + z = 247. \quad (C) \\ x + 0 + 9z = 242 \end{cases}$$

[2] Let the length of the brocade be a, the length of the damask b, and the length of the satin c. From the statement we have $(a^2 - b)^2 + c = 358829, c - b = 2, b - a = 1.$ (G)

[3]					
	7 brocade	1 damask	0 satin	273 silk	(1)
	0 brocade	8 damask	1 satin	247 silk	(2)
	1 brocade	0 damask	69 satin	1842 silk	(3)
(3) × 7,	7 brocade	0 damask	−63 satin	1694 silk	(4)
(1) − (4),	0 brocade	1 damask	−63 satin	−1421 silk	(5)
(5) × 8,		8 damask	−504 satin	−11368 silk	(6)

数又自乘，加绸匹长，共得58829尺。绫匹长比绸匹长少2尺，比锦匹长多1尺。问：锦、绫、绸三色用丝及匹法各为多少？

答：锦2丈5尺，丝35两；

绫2丈6尺，丝28两；

绸2丈8尺，丝23两。

术：如方程术求解，援引正负术，得到锦、绫、绸三种丝每匹的用丝数。 设天元一为锦匹长，以如积方法求其解。得到−358825为常数项，3为一次项系数，−1为二次项系数，−2为三次项系数，1为最高次项系数，开四次方，得到锦匹长。 又：设天元一为绫匹长，以如积方法求其解。得到−358826为常数项，−5为一次项系数，11为二次项系数，−6为三次项系数，1为最高次项系数，开四次方，得到绫匹长。 又：设天元一为绸匹长，以如积方法求其解。得到−358708为常数项，−153为一次项系数，71为二次项系数，−14为三次项系数，1为最高次项系数，开四次方，得到绸匹长。符合所问。

2.

【原文】

今有米、麦、豆共粜得钱三贯四百八文。只云：米取弱半，麦取大半，豆取中半，共得二十八斗。又米取中半，麦取少半，豆取强半，共得三十二斗。又米取强半，麦取中半，豆取大半，共得三十七斗。其米斗价取三分之一，麦斗价取八分之五，豆斗价取二分之一，共得八十七文。又豆、麦斗价和得一百一十文。麦斗价少如米斗价八文。问：三色及斗价各几何？

答曰：米一硕六斗，斗价七十二文；

麦一硕八斗，斗价六十四文；

(2) - (6), 505 satin 11615 silk

satin, 23 *liang* of silk. (C)

[4] The expression in modern form is the equation: $x^4 - 2x^3 - x^2 + 3x - 358825 = 0$.(C)

[5] The expression in modern form is the equation: $y^4 - 6y^3 + 11y^2 - 5y - 358826 = 0$. (C)

[6] The expression in modern form is the equation: $z^4 - 14z^3 + 71z^2 - 153z - 358708 = 0$. (C)

2. Millet, wheat, and beans are sold for 3408 cash. The weak half of the millet, the great half of the wheat, and the middle half of the beans are equal to 28 *dou*; the middle half of the millet, the less half of the wheat, and the great half of the beans are equal to 32 *dou*; and the strong half of the millet, the middle half of the wheat, and the great half of the beans are equal to 37 *dou*. One-third of the price of a *dou* of millet, five-eighths that of a *dou* of wheat, and one-half that of a *dou* of beans amount to 87 cash; and the price of a *dou* of wheat and a *dou* of beans amounts to 110 cash; the price of a *dou* of wheat is less than that of a *dou* of millet by 8 cash. Find the amount of each kind of grain and the price per *dou*.

豆二硕四斗，斗价四十六文。

𝍨	𝍥	𝍢	米
𝍥	𝍣	𝍧	麦
𝍧	𝍨	𝍥	豆
𝍣𝍬𝍣	𝍢𝍰𝍣	𝍢𝍫𝍥	共

术曰：先以合分法[1]求之，次如方程，正负术入之。左行得米，中行得麦，右行得豆。[2]　又三色斗价，如前术求之，得二贯八十八文，即三色共价。[3]　立天元一为米斗价，如积求之。得八百六十四为正实，一十二为益方，上实下法而一，[4]得米斗价七十二文。　又：立天元一为麦斗价，如积求之。得七百六十八为正实，一十二为益方，开无隅平方而一，[5]得麦斗价六十四文。　又立天元一为豆斗价，如积求之。得五百五十二为益实，一十二为从方，上实下法而一，[6]得豆斗价四十六文。合问。

【注释】

[1] 合分指以乘及公倍的方法来变形分数，如以下方法所示：

$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	，由合分法我们得到	9	6	3
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$		6	4	8
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{2}$		8	9	6
37	32	28		444	384	336

表达式的现代表述形式为方程组：

Ans. Millet, 1 shuo 6 *dou*; price per *dou*, 72 cash;

Wheat, 1 shuo 8 *dou*; price per *dou*, 64 cash.

Beans, 2 shuo 4 *dou*; price per *dou*, 46 cash.

Millet	9	6	3
Wheat	6	4	8
Beans	8	9	6
Total	444	384	336

Process. By the *he fen* method [1] and then by the law for the solution of simultaneous equations with positive and negative terms, we obtain from the left column the millet, from the middle the wheat, and from the right the beans. [2] For finding the prices of the three kinds of grain use the same method. We have 2088 cash for the total price. [3] Let the element *tian* be the price of a *dou* of millet. From the statement we have 864 for the positive *shi* and 12 for the negative *fang*, a linear expression.[4] By division we have 72, the price of 1 *dou* of millet. Again let the element *tian* be the price of 1 *dou* of wheat. From the statement we have 768 for the positive *shi*, and 12 for the negative *fang*, a linear expression [5] whose root, 64, is the price of 1 *dou* of wheat. Again let the element *tian* be the price of 1 *dou* of beans. From the statement we have 552 for the negative *shi* and 12 for the positive *fang*, a linear expression[6] whose root, 46, is the price of 1 *dou* of beans.

[Notes]

[1] Composition and division which means removing fractions by multiplying by

$$\begin{cases}9x+6y+8z=444\\6x+4y+9z=384\\3x+8y+6z=336\text{。（陈）}\end{cases}$$

［2］此先用《九章算术》的合分术，求出公分母，再消元，化成：

$$\begin{matrix}x & 0 & 0\\0 & y & 0\\0 & 0 & z\\16 & 18 & 24\end{matrix}$$

。（郭）

［3］仍用合分术，将$\begin{cases}\frac{1}{3}a+\frac{5}{8}b+\frac{1}{2}c=87\\a+b=110\\a-b=8\end{cases}$的第1行化成$8a+15b+12c=2088$。（郭）

［4］开方式的现代形式为：$-12x+864=0$。（陈）

［5］开方式的现代形式为：$-12y+768=0$。（陈）

［6］开方式的现代形式为：$12z-552=0$。（陈）

【今译】

今有米、麦、豆，共粜得钱3贯408文。只知道：取米的$\frac{1}{4}$，麦的$\frac{2}{3}$，豆的$\frac{1}{2}$，共得28斗；又取米的$\frac{1}{2}$，麦的$\frac{1}{3}$，豆的$\frac{3}{4}$，共得32斗；又取米的$\frac{3}{4}$，麦的$\frac{1}{2}$，豆的$\frac{2}{3}$，共得37斗。取米斗价的$\frac{1}{3}$，麦斗价的$\frac{5}{8}$，豆斗价的$\frac{1}{2}$，共得87文。又豆、麦斗价之和是110文。麦斗价比米斗价少8文。问：米、麦、豆三色的斗数及斗价各为多少？

答：米1石6斗，斗价72文；

麦1石8斗，斗价64文；

豆2石4斗，斗价46文。

the common multiple as shown in the following solution:

$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	by the *he fen* method we have	9	6	3
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$		6	4	8
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{2}$		8	9	6
37	32	28		444	384	336

The expressions in modern form are the equations:

$$9x + 6y + 8z = 444$$

$$6x + 4y + 9z = 384$$

$$3x + 8y + 6z = 336.$$ (C)

[2] Firstly, use the *he fen* method in *The Nine Chapters of Mathematical Procedures* to get the common denominator. Secondly, by cancellation it becomes:

$$\begin{array}{ccc} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \\ 16 & 18 & 24. \end{array}$$ (G)

[3] Continue to use the *he fen* method, make the first line of the group of equations

$$\begin{cases} \frac{1}{3}a + \frac{5}{8}b + \frac{1}{2}c = 87 \\ c + b = 110 \\ a - b = 8 \end{cases}$$ become $8a + 15b + 12c = 2088.$ (G)

[4] The expression in modern form is the equation: $-12x + 864 = 0.$ (C)

[5] The expression in modern form is the equation: $-12y + 768 = 0.$ (C)

[6] The expression in modern form is the equation: $12z - 552 = 0.$ (C)

术：先用合分法求之，再如方程术求解，援引正负术，左行得米的斗数，中行得麦的斗数，右行得豆的斗数。又米、麦、豆三种的斗价，也用前面的方法求之，得到2贯88文，就是三色的共价。设天元一为米斗价，以如积方法求其解。得到864为常数项，−12为一次项系数，上面作为实，下面作为法，除之，得到米斗价72文。又：设天元一为麦斗价，以如积方法求其解。得到768为常数项，−12为一次项系数，开无隅平方，除之，得到麦斗价64文。又：设天元一为豆斗价，以如积方法求其解。得到−552为常数项，12为一次项系数，552作为实，12作为法，除之，得到豆斗价46文。符合所问。

3.

【原文】

今有圭田、梯田各一段，共八亩一十五分亩之八。只云：梯取大阔六分之五，小阔取三分之二，为共，减长八分之三，余二十二步。又大阔取三分之一，长取四分之三，为共，减小阔六分之五，余四十步。又小阔取三分之一，长取八分之五，为共，减大阔四分之三，余二十一步。[1]又倍圭长，与圭阔幂等。[2]问：圭田长、阔各几何？

答曰：圭长三十二步，阔八步。

大阔
小阔
梯长
余步

3. There are two pieces of land, one in the form of an isosceles triangle and the other in the form of a trapezoid. The total area is 8 *mu*. From the sum of five-sixths of the great width and two-thirds of the small width of the trapezoid take three-eighths of its length, the remainder is 22 *bu*; from the sum of one-thirds of its great width and three-fourths of its length take five-sixths of the small width, and the remainder is 40 *bu*; from the sum of one-third of its small width and five-eighths of its length take three-fourths of the great width, and the remainder is 21 *bu*. [1] Twice the length of the isosceles triangle is equal to the square of its width.[2] Find the length and the width of the land in the form of the isosceles triangle.

Ans. Length 32 *bu*;
width 8 *bu*.

−18	4	20	great width
8	−10	16	small width
504	9	−9	length of trapezoid
504	480	538	remainder in *bu*.

术曰：先以合分法求之，后如方程，正负术入之。左行得长，中行得小阔，右行得大阔。[3] 又梯积减共积，余为圭积。[4] 立天元一为圭长，如积求之。得三万二千七百六十八为益实，一为正隅，立方开之，[5] 得圭长三十二步。又：立天元一为圭阔，如积求之。得五百一十二为益实，一为从隅，立方开之，[6] 得圭阔八步。合问。

【注释】

[1] 记梯田的大阔、小阔、长分别为 a_1，a_2，b，则题目的筹式相当于线性方程组，

$$\begin{cases}\frac{5}{6}a_1+\frac{2}{3}a_2-\frac{3}{8}b=22\\ \frac{1}{3}a_1-\frac{5}{6}a_2+\frac{3}{4}b=40\end{cases}$$

$$\begin{cases}\frac{5}{6}a_1+\frac{2}{3}a_2-\frac{3}{8}b=22\\ \frac{1}{3}a_1-\frac{5}{6}a_2+\frac{3}{4}b=40\text{。以合分术化成：}\\ -\frac{3}{4}a_1+\frac{1}{3}a_2+\frac{5}{8}b=21\end{cases}\quad\begin{cases}20a_1+16a_2-9b=528\\ 4a_1-10a_2+9b=480\text{。（郭）}\\ -18a_1+8a_2+15b=504\end{cases}$$

[2] 记圭田的长、阔分别为 c，d 此即：$2c=d^2$。（郭）

[3] 此先用《九章算术》的合分术，求出公分母，再消元，化成：

$$\begin{array}{ccc} b & 0 & 0\\ 0 & a_2 & 0\\ 0 & 0 & a_1\\ 64 & 24 & 36\text{。（郭）}\end{array}$$

[4] 梯田面积 $S_1=\frac{1}{2}(a_1+a_2)b=\frac{1}{2}(36+24)\times64=1920$ 步 $=8$ 亩，圭田面积 $S_2=8\frac{8}{15}-8=\frac{8}{15}$ 亩。（郭）

[5] 开方式的现代形式为：$x^3-32768=0$。（陈）

[6] 开方式的现代形式为：$x^3-512=0$。（陈）

【今译】

今各有一块圭田、梯田，总面积为 $8\frac{8}{15}$ 亩。只知道：取梯田大阔的 $\frac{5}{6}$，

Process. By the *he fen* method and then by the law for the solution of simultaneous equations with positive and negative terms, we obtain from the left column the length, from the middle the small width, and from the right the great width of the trapezoid.[3] The area of the isosceles triangle can be obtained by subtracting the area of the trapezoid from the total area given.[4] Let the element *tian* be the length of the isosceles triangle. From the statement we have 32768 for the negative *shi* and 1 for the positive *yu*, a cubic expression[5] whose root, 32, is the required length. Again let the element *tian* be the width of the isosceles triangle. From the statement we have 512 for the negative *shi* and 1 for the positive *yu*, a cubic expression[6] whose root, 8, is the required width.

[Notes]

[1] Let the great width, the small width, and the length of the trapezoid be a_1, a_2, and b respectively. The equations of the problem is the same as a group of linear equations:

$$\begin{cases} \frac{5}{6}a_1 + \frac{2}{3}a_2 - \frac{3}{8}b = 22 \\ \frac{1}{3}a_1 - \frac{5}{6}a_2 + \frac{3}{4}b = 40. \\ -\frac{3}{4}a_1 + \frac{1}{3}a_2 + \frac{5}{8}b = 21 \end{cases} \text{By the } \textit{he fen} \text{ method, it becomes} \begin{cases} 20a_1 + 16a_2 - 9b = 528 \\ 4a_1 - 10a_2 + 9b = 480. \text{ (G)} \\ -18a_1 + 8a_2 + 15b = 504 \end{cases}$$

[2] Let the length and the width of the land in the form of the isosceles triangle be c and d respectively. From the statement we have $c = d^2$. (G)

[3] First, use the *he fen* method in *The Nine Chapters of Mathematical Procedures* to get the common denominator. Secondly, by cancellation we have

$$\begin{matrix} b & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_1 \\ 64. & 24 & 36. \end{matrix} \text{ (G)}$$

小阔的$\frac{2}{3}$，其和减去梯田长的$\frac{3}{8}$，余22步；又取梯田大阔的$\frac{1}{3}$，长的$\frac{3}{4}$，其和减去梯田小阔的$\frac{5}{6}$，余40步；又取梯田小阔的$\frac{1}{3}$，长的$\frac{5}{8}$，其和减去梯田大阔的$\frac{3}{4}$，余21步。又圭田长的2倍，与圭田阔的幂相等。问：圭田的长、阔各为多少？

答：圭田长32步，阔8步。

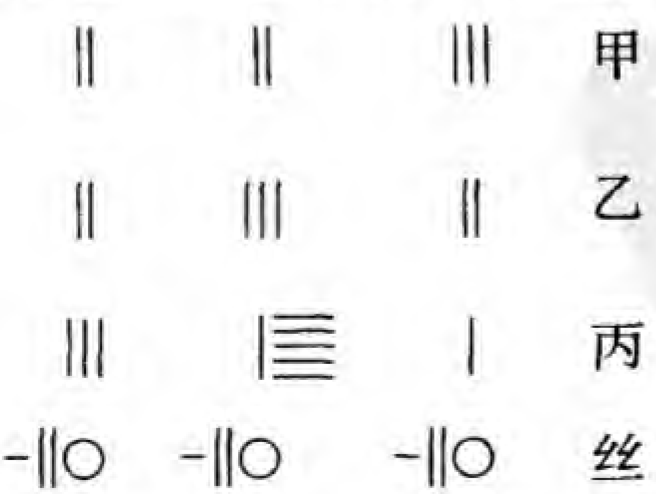
术：先用合分法求之，后如方程术求解，援引正负术，左行得梯田的长，中行得其小阔，右行得其大阔。于是可以求出梯田的面积。又梯田的面积减总面积，余为圭田的面积。 设天元一为圭田的长，以如积方法求其解。得到-32768为常数项，1为最高次项系数，开立方，得到圭田的长32步。 又：设天元一为圭田的阔，以如积方法求其解。得到-512为常数项，1为最高次项系数，开立方，得到圭田的阔8步。符合所问。

4.

【原文】

今有甲、乙、丙买丝，各不知数。甲云得乙丝三分之二，丙丝三分之一，满二斤半。乙云得甲丝三分之二，丙丝二分之一，亦满二斤半。丙云得甲、乙丝各三分之二，亦满二斤半。[1]其丝两价取少半，自乘，内减大半两价，余又自乘，内加大半两价，共得二千八百二十二贯四百八十四文。[2]问：丝及斤价各几何？

答曰：甲一斤半，乙一斤二两，丙一十二两；斤价二贯一十六文。



[4] The area of the land in the form of the trapezoid is

$S_1 = \frac{1}{2}(a_1 + a_2)b = \frac{1}{2}(36 + 24) \times 64 = 1920$ square *bu* = 8 *mu*, and the area of the land in the form of the isosceles triangle is $S_2 = 8\frac{8}{15} - 8 = \frac{8}{15}$ mu. (G)

[5] The expression in modern form is the equation: $x^3 - 32768 = 0$. (C)

[6] The expression in modern form is the equation: $x^3 - 512 = 0$. (C)

4. Jia, Yi, and Bing each bought a certain quantity of silk. Jia says if he gets two-thirds of Yi's and one-third of Bing's he will then have $2\frac{1}{2}$ *jin*; Yi says if he gets two-thirds of Jia's and one-half of Bing's he will then have $2\frac{1}{2}$ *jin*; Bing says if he gets two-thirds of Jia's and two-thirds of Yi's he will then have $2\frac{1}{2}$ *jin*. [1] From the square of the less half of the price of each *liang* take the great half of the price of each *liang*, square the remainder and add the great half of the price of each *liang*; the result is 2822484 cash. [2] Find the amount of silk each bought and the price per *jin*.

Ans. Jia's silk, $1\frac{1}{2}$ *jin*;

Yi's silk, 1 *jin* 2 *liang*;

Bing's silk, 12 *liang*;

price per *jin*, 2016 cash.

术曰：置丝，通两，各以分母乘之，如方程，正负术入之。左行得丙丝，中行得乙丝，右行得甲丝。[3] 立天元一为少半两价，如积求之。得二百八十二万二千四百八十四为益实，二为从方，四为从上廉，四为益下廉，一为正隅，三乘方开之，[4] 得四十二文。以四十八乘之，即斤价。[5] 合问。

【注释】

[1] 记甲、乙、丙买丝分别为a、b、c，则题目的筹式相当于线性方程组，
$\begin{cases} a+\frac{2}{3}b+\frac{1}{3}c=2\frac{1}{2} \\ \frac{2}{3}a+b+\frac{1}{2}c=2\frac{1}{2} \\ \frac{2}{3}a+\frac{2}{3}b+c=2\frac{1}{2} \end{cases}$。以分母乘之，化成：$\begin{cases} 6a+4b+2c=15 \\ 4a+6b+3c=15 \\ 4a+4b+6c=15 \end{cases}$。（郭）

[2] 记丝之两价为d。此即：$[(\frac{1}{3}d)^2-\frac{2}{3}d]^2+\frac{2}{3}d=2822484$。（郭）

[3] 此先用《九章算术》的合分术，求出公分母，再消元，化成：

$$\begin{array}{ccc} c & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & a \\ \frac{3}{4} & 1\frac{1}{8} & 1\frac{1}{2} \end{array}$$
。（郭）

[4] 开方式的现代形式为：$x^4-4x^3+4x^2+2x-2822484=0$。（陈）

[5] 20世纪50年代以前均实行16两为1斤。（郭）

【今译】

今有甲、乙、丙买丝，皆不知道其数量。甲说我若得到乙丝的$\frac{2}{3}$、丙丝的$\frac{1}{3}$，便满$2\frac{1}{2}$斤；乙说我若得到甲丝的$\frac{2}{3}$、丙丝的$\frac{1}{2}$，亦满$2\frac{1}{2}$斤；丙说我若得到甲、乙丝的各$\frac{2}{3}$，亦满$2\frac{1}{2}$斤。取丝两价的$\frac{1}{3}$，自乘，减去丝两价的$\frac{2}{3}$，其余数又自乘，再加丝两价的$\frac{2}{3}$，共得2822贯84文。问：各人买丝及丝斤价各为多少？

2	2	3	Jia' s silk
2	3	2	Yi' s silk
3	1.5	1	Bing' s silk
120	120	120	total

Process. Reduce the *jin* to *liang* then mutiply each term (of the equation) by the denominator. By the rule for simultaneous equations we obtain from the left column Bing' s silk, from the middle Yi' s, and from the right Jia's.[3] Let the element *tian* be the less half of the price of a *liang*. From the statement we have 2822484 for the negative *shi*, 2 for the positive *fang*, 4 for the positive upper *lian*, 4 for the negative lower *lian*, and 1 for the positive *yu*, an expression [4] of the fourth degree whose root is 42. Multiplying this root by 48 we obtain the price per *jin*.[5]

[Notes]

[1] Let the amount of silk Jia bought be a, Yi bought b, and Bing bought c. The equations of the problem are the same as the following group of linear equations:

$$\begin{cases} a+\frac{2}{3}b+\frac{1}{3}c=2\frac{1}{2} \\ \frac{2}{3}a+b+\frac{1}{2}c=2\frac{1}{2} \\ \frac{2}{3}a+\frac{2}{3}b+c=2\frac{1}{2} \end{cases} \quad \text{Multiply it by denominators, and it becomes:} \quad \begin{cases} 6a+4b+2c=15 \\ 4a+6b+3c=15. \\ 4a+4b+6c=15 \end{cases} \text{(G)}$$

[2] Let the silk' s price be d. From the statement we have: $[(\frac{1}{3}d)^2-\frac{2}{3}d]^2+\frac{2}{3}d=2822484$. (G)

[3] Firstly, use the *he fen* method in *The Nine Chapters of Mathematical Procedures* to get the common denominator. Secondly, by cancellation, it becomes:

$$\begin{matrix} c & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & a \\ \frac{3}{4} & 1\frac{1}{8} & 1\frac{1}{2} \end{matrix} \text{. (G)}$$

答：甲买$1\frac{1}{2}$斤，乙1斤2两，丙12两，丝斤价2贯16文。

术：列置丝数化成两，各以分母乘之。后如方程术求解，援引正负术，左行得丙买丝，中行得乙买丝，右行得甲买丝。 设天元一为两价的$\frac{1}{3}$，以如积方法求其解。得到-2822484为常数项，2为一次项系数，4为二次项系数，-4为三次项系数，1为最高次项系数，开四次方，得到42文。以48乘之，就是丝斤价。符合所问。

5.

【原文】

今有三斜田一段。只云：并大斜一，中斜二，减小斜四，余一十五步。又并大斜二，小斜三，减中斜五，少一十五步。又并中斜二，小斜一，减大斜二，余一十五步。[1] 问：中股几何？

答曰：中股三十六步。

术曰：如方程，正负术入之。左行得小斜，中行得中斜，右行得大斜。[2] 立天元一为中股，如积求之。得一千二百九十六为益实，一为正隅，平方开之，[3] 得中股。合问。

【注释】

［1］ 记三斜田的大斜、中斜、小斜分别为a、b、c，则题目的筹式相

[4] The expression in modern form is the equation: $x^4 - 4x^3 + 4x^2 + 2x - 2822484 = 0$. (C)

[5] One *jin* was 16 liang before 1950s. (G)

5. In a piece of land of three oblique sides (scalene triangle) the great side plus twice the middle side minus four times the small side equals 15 *bu*; twice the great side plus three times the small side minus five times the middle side equals 15 *bu*; and twice the middle side plus the small side minus twice the great side equals 15 *bu*. [1] Find the middle leg.

Ans. Middle leg, 36 *bu*.

−2	2	1	great side
2	−5	2	middle side
1	3	−4	small side
15	−15	15	*bu*

Process. By the law for the solution of simultaneous equations with positive and negative terms, we obtain from the left column the small side, from the middle the middle side, and from the right the great side. [2] Let the element *tian* be the middle leg. From the statement we have 1296 for the negative *shi* and 1 for the positive *yu*, a quadratic expression[3] whose root is the required leg.

当于线性方程组，$\begin{cases} a+2b-4c=15 \\ 2a-5b+3c=-15 \\ -2a+2b+c=15 \end{cases}$。（郭）

［2］此即化成：

$$\begin{array}{ccc} c & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & a \\ 45 & 60 & 75 \end{array}$$

。（郭）

［3］开方式的现代形式为：$x^2-1296=0$。（陈）

【今译】

今有一块三斜田。只知道：1倍的大斜与2倍的中斜相加，减4倍的小斜，余15步；又2倍的大斜与3倍的小斜相加，减5倍的中斜，少15步；又2倍的中斜与1倍的小斜相加，减2倍的大斜，余15步。问：中斜的高为多少？

答：中斜的高36步。

术：如方程术求解，援引正负术，左行得小斜，中行得中斜，右行得大斜。　设天元一为中斜的高，以如积方法求其解。得到-1296为常数项，1为最高次项系数，开平方，得到中斜的高。符合所问。

6.

【原文】

今有直田、环田各一段，共一十三亩四分亩之一。[1] 只云：并环田外周一、中周二、实径三，与六个直田斜相较之，多六步。又并外周二、中周一、直斜二，与六十三个实径相较之，少二步。又并外周二、实径五、

[Notes]

[1] Let the great side, the middle side, and the small side of the land be a, b, and c respectively. The equations of the problem are the same as the following group of linear equations:

$$\begin{cases} a + 2b - 4c = 15 \\ 2a - 5b + 3c = -15. \ (\mathrm{G}) \\ -2a + 2b + c = 15 \end{cases}$$

[2] Namely it becomes

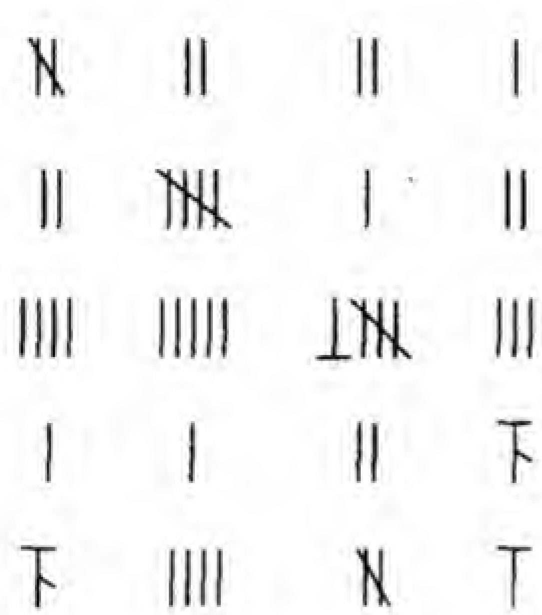
$$\begin{array}{ccc} c & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & a \\ 45 & 60 & 75\,. \ (\mathrm{G}) \end{array}$$

[3] The expression in modern form is the equation: $x^2 - 1296 = 0$. (C)

6. Two piece of land, one in the form of a rectangle and the other a ring, contain $13\frac{1}{4}$ *mu*.[1] The outer circumference, twice the inner circumference, and three times the width of the ring exceed six times the diagonal of the rectangle by 6 *bu*; twice the outer circumference, the inner circumference, and twice the diagonal of the rectangle equal sixty-three times the width of

直斜一，与四个中周相较之，多四步。又并中周二、实径四、直斜一，与二个外周相较之，少六步。[2] 问：直田长、平各几何？

答曰：直田长七十步，阔二十四步。



外周
中周
实经
直斜
步

术曰：如方程，正负术入之。左行得直田斜，次行得实径，次行得中周，右行得外周。[3] 又环积减共积，余为直积。 立天元一为阔，如积求之。得二百八十二万二千四百为正实，五千四百七十六为益上廉，一为正隅，三乘方开之，[4] 得阔。 又：立天元一为长，如积求之。得二百八十二万二千四百为益实，五千四百七十六为从上廉，一为益隅，三乘方开之，[5] 得长。 又：立天元一为和，如积求之。得八千八百三十六为益实，一为正隅，平方开之，[6] 得和。 又：立天元一为较，如积求之。得二千一百一十六为益实，一为正隅，平方开之，[7] 得较。合问。

【注释】

[1] 记直田、环田的面积分别为 S_1，S_2，则 $S_1 + S_2 = 13\frac{1}{4}$。（郭）

[2] 记环田的外、中周、实径，直田斜分别为 l_1，l_2，d，c，则题目的筹式相

the ring less 2 *bu*; twice the outer circumference of the ring, five times its width, and the diagonal of the rectangle exceed four times the inner circumference of the ring by 4 *bu*; twice the inner circumference of the ring, four times the width, and the diagonal of the rectangle equal twice the outer circumference less 6 *bu*. [2] Find the length and the width of the rectangle.

Ans. Length, 70 *bu*; width, 24 *bu*.

−2	2	2	1	outer circumference
2	−4	1	2	inner circumference
4	5	−63	3	width of the ring
1	1	2	−6	diagonal of the rectangle
−6	4	−2	6	*bu*

Process. By the law for the solution of simultaneous equations with positive and negative terms, we obtain from the left column the diagonal of the rectangle, from the second the width of the ring, from the third the inner circumference, and from the fourth the outer circumference. [3] From the total area subtract the area of the ring, and the remainder is the area of the rectangle. Let the element *tian* be the width of the rectangle. From the statement we have 2822400 for the positive *shi*, 5476 for the negative upper *lian*, and 1 for the positive *yu*, an expression [4] of the fourth degree whose root is the required width. Again let the element *tian* be the length. From the statement we have 2822400 for the negative *shi*, 5476 for the positive upper *lian*, and 1 for the negative *yu*, an expression [5] of the fourth degree whose root is the required length. Again let the element *tian* be the

当于线性方程组，$\begin{cases} l_1 + 2l_2 + 3d - 6c = 6 \\ 2l_1 + l_2 - 63d + 2c = -2 \\ 2l_1 - 4l_2 + 5d + c = 4 \\ -2l_1 + 2l_2 + 4d + c = -6 \end{cases}$。（郭）

[3] 通过方程术和正负术化成：

$$\begin{matrix} c & 0 & 0 & 0 \\ 0 & d & l_2 & 0 \\ 0 & 0 & 0 & l_1 \\ 74 & 10 & 120 & 180 \end{matrix}$$

。（郭）

[4] 开方式的现代形式为：$x^4 - 5476x^2 + 2822400 = 0$。（陈）

[5] 开方式的现代形式为：$-x^4 + 5476x^2 - 2822400 = 0$。（陈）

[6] 开方式的现代形式为：$x^2 - 8836 = 0$。（陈）

[7] 开方式的现代形式为：$x^2 - 2116 = 0$。（陈）

【今译】

今有直田、环田各一段，面积共$13\frac{1}{4}$亩。只云：将环田的1倍的外周、2倍的中周、3倍的实径相加，比6倍的直田斜多6步。又将环田的2倍的外周、1倍的中周、2倍的直田斜相加，比63倍的实径少2步。又将环田的2倍的外周、5倍的实径、1倍的直田斜相加，比4倍的中周多4步。又将环田的2倍的中周、4倍的实径、1倍的直田斜相加，比2倍的外周少6步。问：直田的长、阔各为多少？

答：直田长70步，阔24步。

术：如方程术求解，援引正负术，左行得直田斜，第二行得实径，第三行得中周，右行得外周。又环田的面积减总面积，余为直田的面积。　设天元一为直田的阔，以如积方法求其解。得到2822400为常数项，−5476为二次项系数，1为最高次项系数，开四次方，得到直田的阔。　　又：设天元一为直田的长，以如积方法求其解。得到

sum of the width and the length. From the statement we have 8836 for the negative *shi* and 1 for the positive *yu*, a quadratic expression[6] whose root is the required sum. Again let the element *tian* be the difference between the length and the width. From the statement we have 2116 for the negative *shi* and 1 for the positive *yu*, a quadratic expression[7] whose root is the required difference.

[Notes]

[1] Let the area of the land in the form of a rectangle be S_1, and the area of the land in the form of a ring S_2. Then, $S_1 + S_2 = 13\frac{1}{4}$. (G)

[2] Let the outer circumference, the inner circumference, the width of the land in the form of a ring be l_1, l_2, d respectively, and the diagonal of the land in the form of a rectangle c. Then the equations of the problem are the same as the following group of linear equations:

$$\begin{cases} l_1 + 2l_2 + 3d - 6c = 6 \\ 2l_1 + l_2 - 63d + 2c = -2 \\ 2l_1 - 4l_2 + 5d + c = 4 \\ -2l_1 + 2l_2 + 4d + c = -6 \end{cases} \text{. (G)}$$

[3] By the law for the solution of simultaneous equations with positive and negative terms, it becomes:

$$\begin{array}{cccc} c & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & l_2 & 0 \\ 0 & 0 & 0 & l_1 \\ 74 & 10 & 120 & 180. \end{array} \text{(G)}$$

[4] The expression in modern form is the equation: $x^4 - 5476x^2 + 2822400 = 0$. (C)

−2822400为常数项，5476为二次项系数，−1为最高次项系数，开四次方，得到直田的长。 又：设天元一为直田的长阔和，以如积方法求其解。得到−8836为常数项，1为最高次项系数，开平方，得到直田的长阔和。 又：设天元一为直田的长阔差，以如积方法求其解。得到−2116为常数项，1为最高次项系数，开平方，得到直田的长阔差。符合所问。

7.

【原文】

今有勾股田一段，取勾弦和一、股弦和二、勾弦较三为共，内减股弦较四，余二百六十步。又勾弦和二、股弦和一、股弦较三为共，内减勾弦较四，余七十六步。又勾弦和三、勾弦较二、股弦较一为共，内减股弦和二，余五十五步。又股弦和二、勾弦较一、股弦较三为共，内减勾弦和三，余二十八步。[1] 问：勾、股、弦各几何?

答曰：勾一十二步，股三十五步，弦三十七步。

术曰：如方程，正负术人之。左行得股弦较，次行得勾弦较，次行得股弦和，右行得勾弦和。[2] 立天元一为勾，如积求之。得一百四十

[5] The expression in modern form is the equation: $-x^4 + 5476x^2 - 2822400 = 0$. (C)

[6] The expression in modern form is the equation: $x^2 - 8836 = 0$. (C)

[7] The expression in modern form is the equation: $x^2 - 2116 = 0$. (C)

7. A piece of land is in the form of a right triangle. The sum of the hypotenuse and the base, twice the sum of the hypotenuse and the leg, and three times the difference between the hypotenuse and the base exceed four times the difference between the hypotenuse and the leg by 260 *bu*; twice the sum of the hypotenuse and the base, the sum of the hypotenuse and the leg, and three times the difference between the hypotenuse and the leg exceed four times the difference between the hypotenuse and the base by 76 *bu*; three times the sum of the hypotenuse and the base, twice the difference between the hypotenuse and the base , and the difference between the hypotenuse and the leg exceed twice the sum of the hypotenuse and the leg by 55 *bu*; twice the hypotenuse and the leg, the difference between the hypotenuse and the base, and three times the difference between the hypotenuse and the leg exceed three times the sum of the hypotenuse and the base by 28 *bu*.[1] Find the hypotenuse, the leg, and the base of the right triangle.

Ans. Base, 12 *bu*; leg, 35 *bu*; hypotenuse, 37 *bu*.

四为益实，一为正隅，平方开之，[3] 得勾。 立天元一为股，如积求之。得一千二百二十五为益实，一为正隅，平方开之，[4] 得股。 立天元一为弦，如积求之。得一千三百六十九为益实，一为正隅，平方开之，[5] 即弦。合问。

【注释】

[1] 记勾弦和、股弦和、勾弦较、股弦较分别为p，q，r，s，则题目的筹式相当于线性方程组，$\begin{cases} p+2q+3r-4s=260 \\ 2p+q-4r+3s=76 \\ 3p-2q+2r+s=55 \\ -3p+2q+r+3s=28 \end{cases}$。（郭）

[2] 通过方程术和正负术化成：

$$\begin{array}{cccc} s & 0 & 0 & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & q & 0 \\ 0 & 0 & 0 & p \\ 2 & 25 & 72 & 49 \end{array}$$

。（郭）

[3] 开方式的现代形式为：$x^2-144=0$。（陈）

[4] 开方式的现代形式为：$x^2-1225=0$。（陈）

[5] 开方式的现代形式为：$x^2-1369=0$。（陈）

【今译】

今有一勾股田，取1倍的勾弦和、2倍的股弦和、3倍的勾弦较相加，内中减去4倍的股弦较，余260步。取2倍的勾弦和、1倍的股弦和、3倍的股弦较相加，内中减去4倍的勾弦较，余76步。取3倍的勾弦和、2倍的勾弦较、1倍的股弦较相加，内中减去2倍的股弦和，余55步。取2倍的股弦和、1倍的勾弦较、3倍的股弦较相加，内中减去3倍的勾弦

−3	3	2	1	sum of the hypotenuse and the base
2	−2	1	2	sum of the hypotenuse and the leg
1	2	−4	3	difference between the hypotenuse and the base
3	1	3	−4	difference between the hypotenuse and the leg
28	55	76	260	*bu*.

Process. Applying the law for the solution of simultaneous equations with positive and negative terms, we obtain from the left column the difference between the hypotenuse and the leg, from the second the difference between the hypotenuse and the base, from the third the sum of the hypotenuse and the leg, and from the fourth the sum of the hypotenuse and the base.[2] Let the element *tian* be the base. From the statement we have 144 for the negative *shi* and 1 for the positive *yu*, an quadratic expression[3] whose root is the required base. Again let the element *tian* be the leg. From the statement we have 1225 for the negative *shi* and 1 for the positive *yu*, a quadratic expression[4] whose root is the required leg. Again let the element *tian* be the hypotenuse. From the statement we have 1369 for the negative *shi* and 1 for the positive *yu*, a quadratic expression[5] whose root is the required hypotenuse.

[Notes]

[1] Let the sum of the hypotenuse and the base be p, the sum of the hypotenuse and the leg q, the difference between the hypotenuse and the base r, and the difference between the hypotenuse and the leg s. Then the equations of the problem are the same as the following group of linear equations:

$$\begin{cases} p + 2q + 3r - 4s = 260 \\ 2p + q - 4r + 3s = 76 \\ 3p - 2q + 2r + s = 55 \\ -3p + 2q + r + 3s = 28 \end{cases} . \quad (G)$$

和，余 28 步。问：勾、股、弦各为多少？

答：勾 12 步，股 35 步，弦 37 步。

术：如方程术求解，援引正负术，左行得股弦较，第二行得勾弦较，第三行得股弦和，右行得勾弦和。设天元一为勾，以如积方法求其解。得到 −144 为常数项，1 为最高次项系数，开平方，得到勾。设天元一为股，以如积方法求其解。得到 −1225 为常数项，1 为最高次项系数，开平方，得到股。设天元一为弦，以如积方法求其解。得到 −1369 为常数项，1 为最高次项系数，开平方，得到弦。符合所问。

8.

【原文】

今有平圆、立圆、平方、立方各一所。只云：平圆积取九分之一，立圆积取九分之二，平方积取五分之三，减立方积九分之八，盈二尺。又平圆积取九分之一，立方积取九分之二，立圆积取四分之一，减于平方积五分之四，不足二尺。又立圆积取四分之一，立方积取九分之二，平方积取五分之一，减平圆积三分之二，盈二尺。又平方积取五分之一，立方积取九分之四，平圆积取三分之一，减于立圆积九分之七，不足二尺。[1] 其立圆径不及平方面一尺，却多立方面一尺，如平圆径三分之二。[2] 问：四事各几何？

答曰：平圆径六尺，立圆径四尺，

平方面五尺，立方面三尺。

[2] By the law for the solution of simultaneous equations with positive and negative terms, it becomes:

$$\begin{array}{cccc} s & 0 & 0 & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & q & 0 \\ 0 & 0 & 0 & p \\ 2 & 25 & 72 & 49. \end{array} \text{(G)}$$

[3] The expression in modern form is the equation: $x^2 - 144 = 0$. (C)

[4] The expression in modern form is the equation: $x^2 - 1225 = 0$. (C)

[5] The expression in modern form is the equation: $x^2 - 1369 = 0$. (C)

8. There are a circle, a sphere, a square, and a cube. If from the sum of one-ninth of the area of the circle, two-ninths of the volume of the sphere, and three-fifths of the area of the square we subtract eight-ninths of the volume of the cube, then we have 2 *chi* for the remainder. The sum of one-ninth of the area of the circle, two-ninths of the volume of the cube, and one-fourth of the volume of the sphere exceeds four-fifths of the area of the square by 2 *chi*; the sum of one-fourth of the volume of the sphere, two-ninths of the volume of the cube, and one-fifth of the area of the square exceeds two thirds of the area of the circle by 2 *chi*; the sum of one-fifth of the area of the square, four-ninths of the volume of the cube, and one-third of the area of the circle is less than seven-ninths of the volume of the sphere by 2 *chi*. [1] The diameter of the sphere is less by 1 *chi* than a side of the square but exceeds by 1 *chi* a side of the cube and equals two-thirds of the diameter of the circle. [2] Find the four matters.

Ans. Diameter of the circle, 6 *chi*;

diameter of the sphere, 4 *chi*;

术曰：先以合分法求之，次如方程，正负术人之。左行得立方积，次行得平方积，次行得立圆积，右行得平圆积。[3] 并之，为共积。　立天元一为平圆径，如积求之。得二千四百八十四为益实，七十二为从方，三为益廉，十为从隅，立方开之，[4] 得平圆径。　又：立天元一为立圆径，如积求之。得三百六十八为益实，一十六为从方，一为益廉，五为正隅，立方开之，[5] 得立圆径。　又：立天元一为平方面，如积求之。得三百九十为益实，三十三为从方，一十六为益廉，五为从隅，立方开之，[6] 得平方面。　又：立天元一为立方面，如积求之。得三百四十八为益实，二十九为从方，一十四为从廉，五为从隅，立方开之，[7] 得立方面。合问。

【注释】

[1] 记平圆积、立圆积、平方积、立方积分别为 S_1，S_2，S_3，S_4，则题目的筹式相当于线性方程组

$$\begin{cases}\frac{1}{9}S_1+\frac{2}{9}S_2+\frac{3}{5}S_3-\frac{8}{9}S_4=2\\ \frac{1}{9}S_1+\frac{1}{4}S_2-\frac{4}{5}S_3+\frac{2}{9}S_4=-2\\ -\frac{2}{3}S_1+\frac{1}{4}S_2+\frac{1}{5}S_3+\frac{2}{9}S_4=2\\ \frac{1}{3}S_1-\frac{7}{9}S_2+\frac{1}{5}S_3+\frac{4}{9}S_4=-2\end{cases}$$

。以合分术化成：

$$\begin{cases}5S_1+10S_2+27S_3-40S_4=90\\ 20S_1+45S_2-144S_3+40S_4=-360\\ -120S_1+45S_2+36S_3+40S_4=360\\ 15S_1-35S_2+9S_3+20S_4=-90\end{cases}$$

。（郭）

[2] 记平圆径、立圆径、平方面、立方面分别为 d_1，d_2，a_1，a_2 此即：

$a_1-d_2=1$，$d_2-a_2=1$，$d_2=\frac{2}{3}d_1$。（郭）

[3] 此先用《九章算术》的合分术，求出公分母，再消元，化成：

side of the square, 5 *chi*;

side of the cube, 3 *chi*.

15	−110	20	5	circle
−35	45	45	10	sphere
9	36	−144	27	square
20	40	40	−40	cube
−90	360	−360	90	total

Process. Applying the *he fen* method and the law for the solution of simultaneous equations with positive and negative terms, we obtain from the left column the volume of the cube, from the second the area of the square, from the third the volume of the sphere, and from the fourth the area of the circle. [3] Adding we have the *gong ji*. Let the element *tian* be the diameter of the circle. From the statement we have 2484 for the negative *shi*, 72 for the positive *fang*, 3 for the negative *lian*, and 10 for the positive *yu*, a cubic expression[4] whose root is the diameter of the circle. Again let the element *tian* be the diameter of the sphere. From the statement we have 368 for the negative *shi*, 16 for the positive *fang*, 1 for the negative *lian*, and 5 for the positive *yu*, a cubic expression [5] whose root is the diameter of the sphere. Again let the element *tian* be a side of the square. From the statement we have 390 for the negative *shi*, 33 for the positive *fang*, 16 for the negative *lian*, and 5 for the positive *yu*, a cubic expression[6] whose root is a side of the square. Again let the element *tian* be a side of the cube. From the statement we have 348 for the negative *shi*, 29 for the positive *fang*, 14 for the positive *lian*, and 5 for the positive *yu*, a cubic expression [7] whose root is a side of the cube.

$$\begin{array}{cccc} S_4 & 0 & 0 & 0 \\ 0 & S_3 & 0 & 0 \\ 0 & 0 & S_2 & 0 \\ 0 & 0 & 0 & S_1 \\ 27 & 25 & 36 & 27 \end{array}$$
。（郭）

［4］开方式的现代形式为：$10x^3 - 3x^2 + 72x - 2484 = 0$。（陈）
［5］开方式的现代形式为：$5x^3 - x^2 + 16x - 368 = 0$。（陈）
［6］开方式的现代形式为：$5x^3 - 16x^2 + 33x - 390 = 0$。（陈）
［7］开方式的现代形式为：$5x^3 + 14x^2 + 29x - 348 = 0$。（陈）

【今译】

今各有一圆、球、正方形、正方体。只云：取圆面积的$\frac{1}{9}$，球体积的$\frac{2}{9}$，正方形面积的$\frac{3}{5}$，其和减去正方体体积的$\frac{8}{9}$，盈余2尺；又取圆面积的$\frac{1}{9}$，正方体体积的$\frac{2}{9}$，球体积的$\frac{1}{4}$，其和减去正方形面积的$\frac{4}{5}$，不足2尺；又取球体积的$\frac{1}{4}$，正方体体积的$\frac{2}{9}$，正方形面积的$\frac{1}{5}$，其和减去圆面积的$\frac{2}{3}$，盈余2尺；又取正方形面积的$\frac{1}{5}$，正方体体积的$\frac{4}{9}$，圆面积的$\frac{1}{3}$，其和减去球体积的$\frac{7}{9}$，不足2尺。球的直径比正方形边长少1尺，却比正方体边长多1尺，恰等于圆直径的$\frac{2}{3}$。问：圆直径、球直径、正方形边长、正方体边长各为多少？

答：圆直径6尺，球直径4尺，
正方形边长5尺，正方体边长3尺。

术：先用合分法求之，再如方程术求解，援引正负术，左行得正方体体积，第二行得正方形面积，第三行得球体积，右行得圆面积。将它们相加就是共积。　设天元一为圆直径，以如积方法求其解。得到−2484为常数项，72为一次项系数，−3为二次项系数，10为最高次项系数，开立方，得到圆直径。　又：设天元一为球直径，以如积方法求其解。得到−368为常数项，16为一次项系数，−1为二次项系数，5为最高次项系数，开立方，得到球直径。　又：设天元一为正方形边长，以如积方法求其解。得到−390为常数项，33为一次项系数，−16为二次项系数，5为最高次项系数，开立方，得到正方形边长。　又：设天元一为正方体边长，以如积方法求其解。得到−348为常数项，29为一次项系数，14为二次项系数，5为最高次项系数，开立方，得到正方形边长。符合所问。

【Notes】

[1] Let the area of the circle be S_1, the volume of the sphere S_2, the area of the square S_3, and the volume of the cube S_4. Then the equations of the problem are the same as the following group of linear equations:

$$\begin{cases} \frac{1}{9}S_1 + \frac{2}{9}S_2 + \frac{3}{5}S_3 - \frac{8}{9}S_4 = 2 \\ \frac{1}{9}S_1 + \frac{1}{4}S_2 - \frac{4}{5}S_3 + \frac{2}{9}S_4 = -2 \\ -\frac{2}{3}S_1 + \frac{1}{4}S_2 + \frac{1}{5}S_3 + \frac{2}{9}S_4 = 2 \\ \frac{1}{3}S_1 - \frac{7}{9}S_2 + \frac{1}{5}S_3 + \frac{4}{9}S_4 = -2 \end{cases}$$

By the *he fen* method we have

$$\begin{cases} 5S_1 + 10S_2 + 27S_3 - 40S_4 = 90 \\ 20S_1 + 45S_2 - 144S_3 + 40S_4 = -360 \\ -120S_1 + 45S_2 + 36S_3 + 40S_4 = 360 \\ 15S_1 - 35S_2 + 9S_3 + 20S_4 = -90 \end{cases} \text{. (G)}$$

[2] Let the diameter of the circle be d_1, the diameter of the sphere d_2, the side of the square a_1, and the side of the cube a_2. From the statement we have $a_1 - d_2 = 1$, $d_2 - a_2 = 1$, $d_2 = \frac{2}{3} d_1$.(G)

[3] Firstly, use the *he fen* method in *The Nine Chapters of Mathematical Procedures* to get the common denominator. Secondly, by cancellation, it becomes:

$$\begin{matrix} S_4 & 0 & 0 & 0 \\ 0 & S_3 & 0 & 0 \\ 0 & 0 & S_2 & 0 \\ 0 & 0 & 0 & S_1 \\ 27 & 25 & 36 & 27 \end{matrix} \text{. (G)}$$

[4] The expression in modern form is the equation: $10x^3 - 3x^2 + 72x - 2484 = 0$. (C)

[5] The expression in modern form is the equation: $5x^3 - x^2 + 16x - 368 = 0$. (C)

[6] The expression in modern form is the equation: $5x^3 - 16x^2 + 33x - 390 = 0$. (C)

[7] The expression in modern form is the equation: $5x^3 + 14x^2 + 29x - 348 = 0$. (C)

杂范类会 一十三问

1.

【原文】

今有沉香立圆球一只，径十寸。今从顶截周八寸四分。[1] 问：厚几何？

答曰：二分。

术曰：立天元一为截顶厚，如积求之。得一寸九分六厘为正实，一十寸为益方，一寸为从隅，平方开之，[2] 得顶厚。合问。

【注释】

[1] 取 $\pi=3$，记立圆球径、截周、截顶厚、截周的直径分别为 d，l，v，c，截顶厚是截顶弧的矢，截周的直径就是弦，则由《九章算术》勾股锯圆材术 $d=\frac{(\frac{1}{2}c)^2}{v}+v$，$3c=l$。（郭）

[2] 开方式的现代形式为：$x^2-10x+1.96=0$。（陈）

【今译】

今有一只沉香圆球，直径是10寸。今从顶截出球冠的周长为8.4寸。问：截顶厚为多少？

答：2分。

术：设天元一为截顶厚，以如积方法求其解。得到1.96为常数项，−10为一次项系数，1为最高次项系数，开平方，得到截顶厚。符合所问。

Za Fan Lei Hui (Miscellaneous Problems)

13 Problems

1. From the top of a sphere of sandal wood 10 *cun* in diameter a spherical segment is cut whose circumference is 8 *cun* 4 *fen*.[1] Find the thickness of the segment.

Ans. 2 *fen*.

Process. Let the element *tian* be the thickness of the spherical segment. From the statement we have 1 *cun* 9 *fen* 6 *li* for the positive *shi*, 10 *cun* for the negative *fang*, and 1 *cun* for the positive *yu*, a quadratic expression [2] whose root is the required thickness.

[Notes]

[1] Make $\pi = 3$, and let the diameter of the sphere be d, the circumference of the segment l, the thickness of the segment v, and the diameter of the segment c. The thickness is the *shi* of the arc of the segment, and the diameter of the segment is the chord. Then from the *gou gu ju yuan cai* method in *The Nine Chapters of Mathematical Procedures*, we have $\frac{(\frac{1}{2}c)^2}{v} + v = d, 3c = l$. (G)

[2] The expression in modern form is the equation: $x^2 - 10x + 1.96 = 0$. (C)

2.

【原文】

今有人买酒，持钱一十二贯七百四十四文。只云：每瓶纳税八十五文。又共与用钱一百二十六文无钱纳，官准酒九瓶。[1]问：共酒及瓶价各几何？

答曰：共酒三十六瓶，瓶价三百五十四文。

术曰：立天元一为瓶价，如积求之。得一十二万三百六十为益实，一十四为益方，一为正隅，平方开之，[2]得瓶价。合问。

【注释】

[1]记x为每瓶价，则买酒为$\frac{12744}{x}$瓶，共纳税为$\frac{12744}{x}\times 85$，则$\frac{12744}{x}\times 85+126=9x$。（郭）

[2]开方式的现代形式为：$x^2-14x-120360=0$。（陈）

【今译】

今有人带着12贯744文钱买酒。只云：每瓶纳税85文。又共纳税钱及用钱126文都没有钱纳税，官府准折9瓶酒。问：共买的酒的瓶数及每瓶价各为多少？

答：共买酒36瓶，每瓶价354文。

术：设天元一为每瓶价，以如积方法求其解。得到-120360为常数项，-14为一次项系数，1为最高次项系数，开平方，得到每瓶价。符合所问。

3.

【原文】

今有客持珍珠，不知颗数，直银一千二百两。只云：每颗纳税银四钱，

2. A man bought a number of bottles of wine for 12744 cash. The duty on each bottle was 85 cash and the commission 126 cash. The man had no more money with which to pay so the custom-house took 9 bottles of his wine, the cost of which was equivalent to the amount of the charges.[1] Find the number of bottles of wine and the price per bottle.

Ans. Bottles of wine, 36;

price per bottle, 354 cash.

Process. Let the element *tian* be the price per bottle. From the statement we have 120360 for the negative *shi*, 14 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[2] whose root is the required price.

【Notes】

[1] Let x be the price per bottle,then the number of bottles of wine is $\frac{12744}{x}$,and the money for the duty is $\frac{12744}{x} \times 85$.Then, $\frac{12744}{x} \times 85 + 126 = 9x$. (G)

[2] The expression in modern form is the equation: $x^2 - 14x - 120360 = 0$. (C)

3.A merchant owned a certain number of pearls which cost him 1200 taels of silver. The duty on each pearl was 4 *qian*. Since he had no money left he took 7 pearls with which to pay the duty and received 8 taels $2\frac{2}{9}$ *qian* in change.[1] Find how many pearls he owned and the value of each pearl.

准纳七颗，贴与客银八两二钱九分钱之二。[1] 问：元珠及颗价各几何？

答曰：元珠一百三十五颗，颗价八两八钱九分钱之八。

术曰：立天元一为元珠数，如积求之。得一十八万九千为正实，一百八十五为益方，九为益隅，平方开之，[2] 即珠数。 又：立天元一为颗价，如积求之。得四十三万二千为益实，七百四十为益方，六十三为正隅，平方开之，[3] 得颗价。不尽，按之分法求之。合问。

【注释】

[1]记x为珠数，每颗价为$\frac{1200}{x}$，则纳税$\frac{1200}{x}\times 7-8$两$2\frac{2}{9}$钱，因此$\frac{1200}{x}\times 7-8$两$2\frac{2}{9}$钱$=4x$。（郭）

[2]开方式的现代形式为：$-9x^2-185x+189000=0$。（陈）

[3]开方式的现代形式为：$63x^2-740x-432000=0$。（陈）

【今译】

今有客人带着珍珠，不知道颗数，值1200两银。只云：每颗纳税4钱银，准折纳7颗，又贴与客人8两$2\frac{2}{9}$钱银。问：珍珠原来的颗数及每颗价各为多少？

答：原来珍珠135颗，每颗价8两$8\frac{8}{9}$钱。

术：设天元一为原来的珍珠数，以如积方法求其解。得到189000为常数项，-185为一次项系数，-9为最高次项系数，开平方，就是珍珠数。 又：设天元一为珍珠的每颗价，以如积方法求其解。得到-432000为常数项，-740为一次项系数，63为最高次项系数，开平方，得到每颗价。开方不尽，按照之分法求之。符合所问。

Ans. Number of pearls, 135;

value of each pearl, 8 taels $8\frac{8}{9}$ *qian*.

Process. Let the element *tian* be the number of pearls. From the statement we have 189000 for the positive *shi*, 185 for the negative *fang*, and 9 for the negative *yu*, a quadratic expression[2] whose root is the number of pearls.

Again let the element *tian* be the value of each pearl. From the statement we have 432000 for the negative *shi*, 740 for the negative *fang*, and 63 for the positive *yu*, a quadratic expression[3] whose root is the required value. Solving the expression by the *zhi fen* method, since the root is not integral.

[Notes]

[1] Let x be the number of pearls, and the value of each pearl $\frac{1200}{x}$, then the money for the duty is $\frac{1200}{x} \times 7 - 8$ taels $2\frac{2}{9}$ *qian*. Therefore, $\frac{1200}{x} \times 7 - 8$ taels $2\frac{2}{9}$ qian $= 4x$. (G)

[2] The expression in modern form is the equation: $-9x^2 - 185x + 189000 = 0$. (C)

[3] The expression in modern form is the equation: $63x^2 - 740x - 432000 = 0$. (C)

4.

【原文】

今有人赎解，本、利共收九贯八百五十文。只云：利钱平方开之，加入本钱，共得五贯六百九十文。又开方数如日一百二十五分日之一十三。月率三十文，与日同。[1] 问：本、利及日数、每贯月利几何？

答曰：本五贯六百二十五文，每贯月利三十六文七十五分文之四，

二十个月零二十五日，利四贯二百二十五文。

术曰：立天元一为本钱，如积求之。得三千二百三十六万六千二百五十为正实，一万一千三百七十九为益方，一为正隅，平方开之，[2] 即本钱。余依法求之。合问。

【注释】

[1] 记本钱、利钱、日数分别为a，p，r，此即：$a+p=9850$，$\sqrt{p}+a=5690$，$\sqrt{p}=\frac{13}{125}r$。（郭）

[2] 开方式的现代形式为：$x^2-11379x+32366250=0$。（陈）

【今译】

今有人赎当，本钱和利钱共收到9贯850文。只云：将利钱开平方，加入本钱，共得5贯690文。又上面的开方数等于日数的$\frac{13}{125}$。月率30文，与日率相同。问：本钱、利钱及日数、每贯的月利各为多少？

答：本钱5贯625文，每贯的月利$36\frac{4}{75}$文，

日数为20个月零25日，利钱4贯225文。

术：设天元一为本钱，以如积方法求其解。得到32366250为常数项，-11379为一次项系数，1为最高次项系数，开平方，就是本钱。其他各项依法求之。符合所问。

4. A man in order to receive certain things from a pawn-shop has to pay for money received together with interest 9850 cash. The money received and the square root of the interest amount to 5690 cash. The square root of the interest is $\frac{13}{125}$ of the time in days [30 days in a month].[1] Find the amount received, and the interest, the time, and the monthly rate of interest per thousand cash.

Ans. Amount received, 5625 cash;

monthly rate of interest, $36\frac{4}{75}$ cash per thousand;

time, 20 months 25 days;

interest, 4225 cash.

Process. Let the element *tian* be the money received. From the statement we have 32366250 for the positive *shi*, 11379 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[2] whose root is the required amount. The other values can be obtained from this value.

[Notes]

[1] Let the principal be a, the interest p, and the time in days r. From the statement we have $a + p = 9850, \sqrt{p} + a = 5690, \sqrt{p} = \frac{13}{125} r$. (G)

[2] The expression in modern form is the equation: $x^2 - 11379x + 32366250 = 0$. (C)

5.

【原文】

今有钱八贯六百一十二文，已令五人分之。只云：乙如甲五分之三；丙不及乙一贯八百八十文；并甲、丙，以乙除之，所得与戊同；丁少如丙七百九十文。[1] 问：各分几何?

答曰：甲四贯七百文；乙二贯八百二十文；

丙九百四十文；丁一百五十文；

戊二文。

术曰：立天元一为一分之率，如积求之。得九百四十为益实，一万九千七百三十九为益方，二十一为从隅，平方开之，得九百四十文，为一分之率。[2] 合问。

【注释】

[1] 记五人所分钱分别为 a，b，c，d，e，此即：$a+b+c+d+e=8612$，$b=\frac{3}{5}a$，$b-c=1880$，$\frac{a+c}{b}=e$，$c-d=790$。（郭）

[2] “一分之率”即甲所得的$\frac{1}{5}$。开方式的现代形式为：$21x^2-19739x-940=0$。（陈）

【今译】

今有钱8贯612文，已经令五人分之。只云：乙是甲的$\frac{3}{5}$；丙比乙少1贯880文；甲与丙相加，以乙除之，其得数与戊相等；丁比丙少790文。问：五人各分到多少?

答：甲分得4贯700文，乙分得2贯820文，

丙分得940文，丁分得150文，戊分得2文。

术：设天元一为一分之率，以如积方法求其解。得到−940为常数项，−19739为一次项系数，21为最高次项系数，开平方，得到940文，为一分之率。符合所问。

5. Among five persons 8612 cash were divided. Yi received three-fifths as much as Jia; Bing received 1880 cash less than Yi; The sum of Jia's and Bing's shares divided by Yi's equals Wu's; Ding's share is less than Bing's by 790 cash.[1] Find the amount of money each person received.

Ans. Jia, 4700 cash;

Yi, 2820 cash;

Bing, 940 cash;

Ding, 150 cash;

Wu, 2 cash.

Process. Let the element *tian* be the common ratio. From the statement we have 940 for the negative *shi*, 19739 for the negative *fang*, and 21 for the positive *yu*, a quadratic expression [2] whose root, 940, is the required number.

[Notes]

[1] Let the amount of money each person received be a, b, c, d, and e respectively. From the statement we have $a + b + c + d + e = 8612$, $b = \frac{3}{5}a$, $b - c = 1880$, $\frac{a + c}{b} = e$, $c - d = 790$. (G)

[2] The common ratio is one-fifth of Jia received. The expression in modern form is the equation: $21x^2 - 19739x - 940 = 0$. (C)

6.

【原文】

今有木圆球一只，径一尺八寸。欲令漆之，先用布韈。布阔二尺。[1]问：用布长几何？

答曰：三尺六寸二十分寸之九。

术曰：立天元一为布长，如积求之。得七尺二寸九分为益实，二尺为从方，上实，下法而一，[2]即长。合问。

【注释】

[1] 韈，本义是衬托鞍的垫子。（郭）

[2] 开方式的现代形式为：$2x-7.29=0$。（陈）

【今译】

今有一只木圆球，直径是1尺8寸。欲给它上漆，先用布做衬垫，布阔2尺。问：用布长为多少？

答：3尺$6\frac{9}{20}$寸。

术：设天元一为用布长，以如积方法求其解。得到-7.29为常数项，2为一次项系数，上作为实，下作为法，除之，就是用布长。符合所问。

7.

【原文】

今有立方，面五尺。[1]问：东南上角直至西北下角长几何？

答曰：八尺六寸一百七十三分寸之四。

术曰：立天元一为斜长，如积求之。得七十四为益实，一为正隅，平方开之，[2]不尽，命分。合问。

6. A wooden sphere, 1 *chi* 8 *cun* in diameter, which is to be lacquered is first covered with a layer of cloth 2 *chi* wide. [1] Find the length of the cloth.

Ans. 3 *chi* $6\frac{9}{20}$ *cun*.

Process. Let the element *tian* be the length of the cloth. From the statement we have 7 *chi* 2 *cun* 9 *fen* for the negative *shi* and 2 *chi* for the positive *fang*, a linear expression [2] whose root, obtained by division, is the required length.

[Notes]

[1] The character *zhan* appeares in Chinese text. Its original meaning is a cushion used as a foil to a saddle. (G)

[2] The expression in modern form is the equation: $2x - 7\frac{29}{100} = 0$. (C)

7. A side of a cube is 5 *chi*. [1] Find the diagonal from the upper southeast corner to the lower northwest corner.

Ans. 8 *chi* $6\frac{4}{173}$ *cun*.

Process. Let the element *tian* be the length of the diagonal. From the statement we have 74 for the negative *shi* and 1 for the positive *yu*, a quadratic expression[2] whose root, a mixed number, is the required length.

【注释】

[1] 取$\sqrt{2}=\frac{7}{5}$。记正方体的边长、对角线分别为a，l，则$l=\sqrt{(\sqrt{2}a)^2+a^2}=\sqrt{7^2+5^2}$。(郭)

[2] 开方式的现代形式为：$x^2-74=0$。(陈)

【今译】

今有正方体，边长5尺。问：东南上角至西北下角的斜长为多少？

答：8尺$6\frac{4}{173}$寸。

术：设天元一为斜长，以如积方法求其解。得到−74为常数项，1为最高次项系数，开平方。开方不尽，命名一个分数。符合所问。

8.

【原文】

今有圆材，径三尺。只云锯深三寸。[1] 问：锯道长几何？

答曰：一尺八寸。

术曰：立天元一为锯道长，如积求之。得三百二十四为益实，一为正隅，平方开之。[2] 合问。

【注释】

[1] 记圆材径、锯深、锯道长分别为d,v,c，由《九章算术》勾股锯圆材术，$\frac{(\frac{1}{2}c)^2}{v}+v=d$。本题求锯道，下题求锯深，均本此。(郭)

[2] 开方式的现代形式为：$x^2-324=0$。(陈)

【今译】

今有圆材，直径为3尺。只云锯深为3寸。问：锯道长为多少？

答：1尺8寸。

术：设天元一为锯道长，以如积方法求其解。得到−324为常数项，1为最高次项系数，开平方。符合所问。

[Notes]

[1] Make $\sqrt{2}=\frac{7}{5}$, and let the side and the diagonal of the cube be a and l respectively. Then, $l=\sqrt{(\sqrt{2}a)^2+a^2}=\sqrt{7^2+5^2}$. (G)

[2] The expression in modern form is the equation: $x^2 - 74 = 0$. (C)

8. A cross-section 3 *cun* deep is cut into a round piece of timber 3 *chi* in diameter. [1] Find the arc made by the saw.

Ans. 1 *chi* 8 *cun*.

Process. Let the element *tian* be the arc cut by the saw. From the statement we have 324 for the negative *shi* and 1 for the positive *yu*, a quadratic expression[2] whose root is the required arc.

[Notes]

[1] Let the diameter of the timber be d, the depth of the cut v, and the arc made by the saw c. From the method of the *gou gu ju yuan cai* method in *The Nine Chapters of Mathematical Procedures*, we have $\frac{(\frac{1}{2}c)^2}{v}+v=d$. This problem is to find the arc made by the saw.The next problem is to find the depth of the cut.The two problems are based on it. (G)

[2] The expression in modern form is the equation: $x^2 - 324 = 0$. (C)

9.

【原文】

今有圆材，径三尺。只云锯道长一尺八寸。问：深几何？

答曰：三寸。

术曰：立天元一为锯深，如积求之。得八十一为正实，三十为益方，一为正隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$x^2 - 30x + 81 = 0$。（陈）

【今译】

今有圆材，直径为3尺。只云锯道长为1尺8寸。问：锯深为多少？

答：3寸。

术：设天元一为锯深，以如积方法求其解。得到81为常数项，−30为一次项系数，1为最高次项系数，开平方。符合所问。

10.

【原文】

今有天上雁三群，地上雁一群，共三百一只。只云：头群、次群共二百五十六只。又以次群除头群，所得加地上雁，与末群同。地上雁不及次群四十三只。[1] 问：四群各几何？

答曰：头群一百九十二只，次群六十四只，

末群二十四只，地上二十一只。

术曰：立天元一为次群雁，如积求之。得一百二十八为正实，六十六为益方，一为正隅，平方开之，[2] 得次群雁。合问。

9. A cross-section is cut into a round piece of timber 3 *chi* in diameter. The arc of the section is 1 *chi* 8 *cun*. How deep is the cut?

Ans. 3 *cun*.

Process. Let the element *tian* be the depth of the cut. From the statement we have 81 for the positive *shi*, 30 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[1] whose root is the required depth.

[Notes]

[1] The expression in modern form is the equation: $x^2 - 30x + 81 = 0$. (C)

10. Three flocks of geese are flying in the sky and another flock is on the ground. The total number of geese is 301. The number of geese in the first and second flocks is 256; the quotient of those in the first flock by those in the second increased by the flock on the ground is equal to the last flock in the sky; the flock on the ground is less than the second flock in the sky by 43.[1] Find the number of geese in each flock.

Ans. First flock, 192;

second flock, 64;

third flock, 24;

flock on the ground, 21.

【注释】

[1] 记头、次、末、地上雁分别为a, b, c, d, 此即: $a+b+c+d=301$, $a+b=256$, $\frac{a}{b}+d=c$, $b-d=43$。(郭)

[2] 开方式的现代形式为: $x^2-66x+128=0$。(陈)

【今译】

今有天上有三群雁，地上有一群雁，共有301只。只云：头群、次群共有256只；又以次群除头群，所得数加地上雁，与末群相等；地上雁比次群少43只。问：四群雁各为多少？

答：头群192只，次群64只，

末群24只，地上21只。

术：设天元一为次群雁，以如积方法求其解。得到128为常数项，-66为一次项系数，1为最高次项系数，开平方，便得次群雁。符合所问。

11.

【原文】

今有徽术弧田一亩一百七十三步。只云矢不及弦五十步。[1] 问：弦、矢各几何？

答曰：弦六十步，矢一十步。

术曰：立天元一为弦，如积求之。得三十三万四千八百为正实，三万为益方，四百七为从隅，平方开之，[2] 得弦。合问。

【注释】

[1] 记弧田的矢、弦分别为v, c, 据罗士琳云，徽术弧田面积为

$S=\frac{1}{2}[(c+v)v+\frac{7c^2}{200}]=413$。又$c-v=50$。(郭)

[2] 开方式的现代形式为: $407x^2-30000x+334800=0$。(陈)

Process. Let the element *tian* be the number of geese in the second flock. From the statement we have 128 for the positive *shi*, 66 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[2] whose root is the number of geese in the second flock.

[Notes]

[1] Let the first flock of geese be a, the second flock b, the last flock c, and the flock on the ground d. From the statement we have $a + b + c + d = 301$, $a + b = 256$, $\frac{a}{b} + d = c$, $b - d = 43$. (G)

[2] The expression in modern form is the equation: $x^2 - 66x + 128 = 0$. (C)

11. A piece of land in the form of a segment of a circle contains 1 *mu* 173 *bu*. Its *shi* is less than its chord by 50 *bu*.[1] Find the *shi* and the chord using Hui's value (of π).

Ans. Chord, 60 *bu*; *shi*, 10 *bu*.

Process. Let the element *tian* be the chord. From the statement we have 334800 for the positive *shi*, 30000 for the negative *fang*, and 407 for the positive *yu*, a quadratic expression[2] whose root is the required chord.

[Notes]

[1] Let the *shi* and the chord of the land be v and c respectively. According to what Luo Shilin said, the area of the land using Hui's value (of π) is as follows:

$S = \frac{1}{2}[(c + v)v + \frac{7c^2}{200}] = 413$. And, $c - v = 50$. (G)

【今译】

今有依徽术计算的弧田面积1亩173步。只云矢比弦少50步。问：弦、矢各为多少？

答：弦60步，矢10步。

术：设天元一为弦，以如积方法求其解。得到334800为常数项，−30000为一次项系数，407为最高次项系数，开平方，便得弦。符合所问。

12.

【原文】

今有密率弧田积一百三十六步半。只云矢幂多于弦二十一步。[1]问：弦、矢各几何？

答曰：矢七步，弦二十八步。

术曰：立天元一为矢，如积求之。得七千二百三为益实，五百八十八为益方，一十四为益上廉，二十八为从下廉，一为正隅，三乘方开之，[2]得矢七步。合问。

【注释】

［1］据罗士琳云，密率弧田面积为：

$S=\frac{1}{2}\left[(c+v)v+\frac{c^2}{28}\right]=136\frac{1}{2}$。又 $v^2-c=21$。（郭）

［2］开方式的现代形式为：$x^4+28x^3-14x^2-588x-7203=0$。（陈）

【今译】

今有依密率计算的弧田面积$136\frac{1}{2}$步。只云矢幂比弦多21步。问：弦、矢各为多少？

答：矢7步，弦28步。

术：设天元一为矢，以如积方法求其解。得到−7203为常数项，−588为一次项系数，−14为二次项系数，28为三次项系数，1为最高次项系数，开四次方，便得矢7步。符合所问。

[2] The expression in modern form is the equation: $407x^2 - 30000x + 334800 = 0$. (C)

12. A piece of land in the form of a segment of a circle contains $136\frac{1}{2}$ *bu*. The square of the *shi* exceeds the chord by 21 *bu*.[1] Find the chord and the *shi* using the *mi* value (of π).

Ans. *Shi*, 7 *bu*; chord, 28 *bu*.

Process. Let the element *tian* be the length of the *shi*. From the statement we have 7203 for the negative *shi*, 588 for the negative *fang*, 14 for the negative upper *lian*, 28 for the positive lower *lian*, and 1 for the positive *yu*, an expression[2] of the fourth degree whose root, 7 *bu*, is the length of the *shi*.

[Notes]

[1] According to what Luo Shilin said, the area of the land using the *mi* value (of π) is as follows: $S = \frac{1}{2}[(c + v)v + \frac{c^2}{28}] = 136\frac{1}{2}$. And, $v^2 - c = 21$. (G)

[2] The expression in modern form is the equation: $x^4 + 28x^3 - 14x^2 - 588x - 7203 = 0$. (C)

13.

【原文】

今有立方、立圆、平方各一所，共积二万九千九百八十四尺。只云立圆径如立方面七分之六；平方面如立圆径三分之二。[1] 问：三事各几何？

答曰：立圆径二十四尺，立方面二十八尺，平方面一十六尺。

术曰：立天元一为立圆径，如积求之。得一千二百九十五万三千八十八为益实，一百九十二为从廉，九百二十九为从隅，立方开之，[2] 得立圆径。合问。

【注释】

[1] 记立方面、立圆径、平方面分别为 a_1，d，a_2，此即：$a_1^3+\frac{9}{16}d^3+a_2^2=29984$；并且 $d=\frac{6}{7}a_1$，$a_2=\frac{2}{3}d$。（郭）

[2] 开方式的现代形式为：$929x^3+192x^2-12953088=0$。（陈）

【今译】

今有正方体、球、正方形各有一所，共积 29984 尺。只云：球的直径等于正方体边长的$\frac{6}{7}$，正方形的边长是球直径的$\frac{2}{3}$。问：球直径、正方体边长、正方形面积各为多少？

答：球直径 24 尺，正方体边长 28 尺，正方形边长 16 尺。

术：设天元一为球直径，以如积方法求其解。得到 −12953088 为常数项，192 为二次项系数，929 为最高次项系数，开立方，便得到球直径。符合所问。

13. We have a cube, a sphere, and a square. The volumes of the first two and the area of the last equal 29984 *chi*. The diameter of the sphere is six-sevenths of a side of the cube; a side of the square is two-thirds of the diameter of the sphere. [1] Find the three matters [the diameter of the sphere, a side of the cube, and a side of the square].

Ans. Diameter of the sphere, 24 *chi*;

side of the cube, 28 *chi*;

side of the square, 16 *chi*.

Process. Let the element *tian* be the diameter of the sphere. From the statement we have 12953088 for the negative *shi*, 192 for the positive *lian*, and 929 for the positive *yu*, a cubic expression [2] whose root is the required diameter.

[Notes]

[1] Let the side of the cube be a_1, the diameter of the sphere d, and the side of the square a_2. From the statement we have $a_1^3 + \frac{9}{16} d^3 + a_2^2 = 29984$; and, $d = \frac{6}{7} a_1$, $a_2 = \frac{2}{3} d$. (G)

[2] The expression in modern form is the equation: $929x^3 + 192x^2 - 12953088 = 0$. (C)

两仪合辙 一十二问

1.

【原文】

今有勾股积三十步。只云勾股和一十七步。[1] 问：勾弦和几何？

答曰：一十八步。

术曰：立天元一为勾弦和，地元一为勾，天、地配合求之。得三千六百为益实，三千七百六为益方，七十一为益上廉，三十四为从下廉，一为益隅，三乘方开之。[2] 合问。

【注释】

[1] 此即：$\frac{1}{2}ab = 30$，$a + b = 17$。（郭）

[2] 开方式的现代形式为：$-x^4 + 34x^3 - 71x^2 - 3706x - 3600 = 0$。（陈）

【今译】

今有勾股形，面积为 30 步。只云勾股和为 17 步。问：勾弦和为多少？

答：18 步。

术：设天元一为勾弦和，地元一为勾，天元与地元相配合以求其解。得到 −3600 为常数项，−3706 为一次项系数，−71 为二次项系数，34 为三次项系数，1 为最高次项系数，开四次方。符合所问。

2.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：股弦和几何？

Liang Yi He Zhe (Expressions in Two Unknown Elements)

12 Problems

1. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu* . [1] Find the sum of the *gou* and the *xian*.

Ans. 18 *bu*.

Process. Let the element *tian* be the sum of the *gou* and the *xian*, and the element *di* the *gou*. Using these elements we obtain 3600 for the negative *shi*, 3706 for the negative *fang*, 71 for the negative upper *lian*, 34 for the positive lower *lian*, and 1 for the negative *yu*, an expression[2] of the fourth degree whose root is the required sum.

[Notes]

[1] That is, $\frac{1}{2}ab = 30, a + b = 17$. (G)

[2] The expression in modern form is the equation: $-x^4 + 34x^3 - 71x^2 - 3706x - 3600 = 0$. (C)

2.The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the sum of the *gu* and the *xian*.

Ans. 25 *bu*.

答曰：二十五步。

术曰：立天元一为股弦和，地元一为股，天、地配合求之。得三千六百为正实，三千七百六为从方，七十一为从上廉，三十四为益下廉，一为正隅，三乘方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$x^4-34x^3+71x^2+3706x+3600=0$。(陈)

【今译】

今有勾股形，面积为30步。只云勾股和17步。问：股弦和为多少？

答：25步。

术：设天元一为股弦和，地元一为股，天元与地元相配合以求其解。得到3600为常数项，3706为一次项系数，71为二次项系数，−34为三次项系数，1为最高次项系数，开四次方。符合所问。

3.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：弦和和几何？

答曰：三十步。

术曰：立天元一为弦和和，地元一为勾，天、地配合求之。得一百二十为正实，三十四为益方，一为正隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$x^2-34x+120=0$。(陈)

Process. Let the element *tian* be the sum of the *gu* and the *xian*, and the element *di* the *gu*. Using these elements we obtain 3600 for the positive *shi*, 3706 for the positive *fang*, 71 for the positive upper *lian*, 34 for the negative lower *lian*, and 1 for the positive *yu*, an expression[1] of the fourth degree whose root is the required sum.

[Notes]

[1] The expression in modern form is the equation: $x^4 - 34x^3 + 71x^2 + 3706x + 3600 = 0$. (C)

3. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the sum of the *xian* and the *he*.

Ans. 30 *bu*.

Process. Let the element *tian* be the sum of the *xian* and the *he*, and the element *di* the *gou*. Using these elements we obtain 120 for the positive *shi*, 34 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[1] whose root is the required sum.

[Notes]

[1] The expression in modern form is the equation: $x^2 - 34x + 120 = 0$. (C)

【今译】

今有勾股形，面积为30步。只云勾股和17步。问：弦与勾股和之和为多少？

答：30步。

术：设天元一为弦与勾股和之和，地元一为勾，天元与地元相配合以求其解。得到120为常数项，-34为一次项系数，1为最高次项系数，开平方。符合所问。

4.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：弦较和几何？

答曰：二十步。

术曰：立天元一为弦较和，地元一为较，天、地配合求之。得一万四千四百为益实，四百三十六为从上廉，一为益隅，三乘方开之。[1]合问。

【注释】

[1] 开方式的现代形式为：$-x^4+436x^2-14400=0$。（陈）

【今译】

今有勾股形，面积为30步。只云勾股和17步。问：弦与勾股较之和为多少？

答：20步。

术：设天元一为弦与勾股较之和，地元一为勾股较，天元与地元相配合以求其解。得到-14400为常数项，436为二次项系数，-1为最高次项系数，开四次方。符合所问。

4. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the sum of the *xian* and the *jiao*.

Ans. 20 *bu*.

Process. Let the element *tian* be the sum of the *xian* and the *jiao*, and the element *di* the *jiao*. Using these elements we obtain 14400 for the negative *shi*, 436 for the positive upper *lian*, and 1 for the negative *yu*, an expression[1] of the fourth degree whose root is the required sum.

【Notes】

[1] The expression in modern form is the equation: $-x^4 + 436x^2 - 14400 = 0$. (C)

5.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：勾股较几何？

答曰：七步。

术曰：立天元一为勾股较，地元一为勾，天、地配合求之。得四十九为益实，一为正隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$x^2 - 49 = 0$。（陈）

【今译】

今有勾股形，面积为30步。只云勾股和17步。问：勾股较为多少？

答：7步。

术：设天元一为勾股较，地元一为勾，天元与地元相配合以求其解。得到 −49 为常数项，1为最高次项系数，开平方。符合所问。

6.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：勾弦较几何？

答曰：八步。

术曰：立天元一为勾弦较，地元一为勾，天、地配合求之。得三千六百为益实，三千七百六为从方，七十一为益上廉，三十四为益下廉，一为益隅，三乘方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$-x^4 - 34x^3 - 71x^2 + 3706x - 3600 = 0$。（陈）

5. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the difference between the *gu* and the *gou*.

Ans. 7 *bu*.

Process. Let the element *tian* be the difference between the *gu* and the *gou*, and the element *di* the *gou*. Using these elements we obtain 49 for the negative *shi* and 1 for the positive *yu*, a quadratic expression [1] whose root is the required difference.

[Notes]

[1] The expression in modern form is the equation: $x^2 - 49 = \dot{0}$. (C)

6. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the difference between the *xian* and the *gou*.

Ans. 8 *bu*.

Process. Let the element *tian* be the difference between the *xian* and the *gou*, and the element *di* the *gou*. Using these elements we obtain 3600 for the negative *shi*, 3706 for the positive *fang*, 71 for the negative upper *lian*, 34 for the negative lower *lian*, and 1 for the negative *yu*, an expression [1] of the fourth degree whose root is the required difference.

【今译】

今有勾股形，面积为30步。只云勾股和17步。问：勾弦较为多少？

答：8步。

术：设天元一为勾弦较，地元一为勾，天元与地元相配合以求其解。得到 −3600 为常数项，3706 为一次项系数，−71 为二次项系数，−34 为三次项系数，−1 为最高次项系数，开四次方。符合所问。

7.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：股弦较几何？

答曰：一步。

术曰：立天元一为股弦较，地元一为股，天、地配合求之。得三千六百为正实，三千七百六为益方，七十一为从上廉，三十四为从下廉，一为正隅，三乘方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$x^4+34x^3+71x^2-3706x+3600=0$。（陈）

【今译】

今有勾股形，面积为30步。只云勾股和17步。问：股弦较为多少？

答：1步。

术：设天元一为股弦较，地元一为股，天元与地元相配合以求其解。得到3600为常数项，−3706为一次项系数，71为二次项系数，34为三次项系数，1为最高次项系数，开四次方。符合所问。

【Notes】

[1] The expression in modern form is the equation: $-x^4 - 34x^3 - 71x^2 + 3706x - 3600 = 0$. (C)

7. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the difference between the *xian* and the *gu*.

Ans. 1 *bu*.

Process. Let the element *tian* be the difference between the *xian* and the *gu*, and the element *di* the *gu*. Using these elements we obtain 3600 for the positive *shi*, 3706 for the negative *fang*, 71 for the positive upper *lian*, 34 for the positive lower *lian*, and 1 for the positive *yu*, an expression [1] of the fourth degree whose root is the required difference.

【Notes】

[1] The expression in modern form is the equation: $x^4 + 34x^3 + 71x^2 - 3706x + 3600 = 0$. (C)

8.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：弦和较几何？

答曰：四步。

术曰：立天元一为弦和较，地元一为勾，天、地配合求之。得一百二十为正实，三十四为益方，一为从隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$x^2-34x+120=0$。（陈）

【今译】

今有勾股形，面积为30步。只云勾股和17步。问：弦与勾股和之差为多少？

答：4步。

术：设天元一为弦与勾股和之差，地元一为勾，天元与地元相配合以求其解。得到120为常数项，−34为一次项系数，1为最高次项系数，开平方。符合所问。

9.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：弦较较几何？

答曰：六步。

术曰：立天元一为弦较较，地元一为较，天、地配合求之。得一万四千四百为正实，四百三十六为益上廉，一为从隅，三乘方开之。[1] 合问。

8. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the difference between the *he* and the *xian*.

Ans. 4 *bu*.

Process. Let the element *tian* be the difference between the *he* and the *xian*, and the element *di* the *gou*. Using these elements we obtain 120 for the positive *shi*, 34 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[1] whose root is the required difference.

【Notes】

[1] The expression in modern form is the equation: $x^2 - 34x + 120 = 0$. (C)

9. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the difference between the *xian* and the *jiao*.

Ans. 6 *bu*.

Process. Let the element *tian* be the difference between the *xian* and the *jiao*, and the element *di* the *jiao*. Using these elements we obtain 14400 for the positive *shi*, 436 for the negative upper *lian*, and 1 for the positive *yu*, an expression[1] of the fourth degree whose root is the required difference.

【注释】

[1] 开方式的现代形式为：$x^4-436x^2+14400=0$。（陈）

【今译】

今有勾股形，面积为30步。只云勾股和17步。问：弦与勾股差之差为多少？

答：6步。

术：设天元一为弦与勾股差之差，地元一为勾股差，天元与地元相配合以求其解。得到14400为常数项，−436为二次项系数，1为最高次项系数，开四次方。符合所问。

10.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：勾幂几何？

答曰：二十五步。

术曰：立天元一为勾幂，地元一为勾，天、地配合求之。得三千六百为正实，一百六十九为益方，一为正隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$x^2-169x+3600=0$。（陈）

【今译】

今有勾股形，面积为30步。只云勾股和17步。问：勾幂为多少？

答：25步。

术：设天元一为勾幂，地元一为勾，天元与地元相配合以求其解。得到3600为常数项，−169为一次项系数，1为最高次项系数，开平方。符合所问。

【 Notes 】

[1] The expression in modern form is the equation: $x^4 - 436x^2 + 14400 = 0$. (C)

10. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the square of the *gou*.

Ans. 25 *bu*.

Process. Let the element *tian* be the square of the *gou* and the element *di* the *gou*. Using these elements we obtain 3600 for the positive *shi*, 169 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[1] whose root is the square of the *gou*.

【 Notes 】

[1] The expression in modern form is the equation: $x^2 - 169x + 3600 = 0$. (C)

11.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：股幂几何？

答曰：一百四十四步。

术曰：立天元一为股幂，地元一为勾，天、地配合求之。得三千六百为益实，一百六十九为从方，一为益隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$-x^2+169x-3600=0$。（陈）

【今译】

今有勾股形，面积为30步。只云勾股和17步。问：股幂为多少？

答：144步。

术：设天元一为股幂，地元一为勾，天元与地元相配合以求其解。得到−3600为常数项，169为一次项系数，−1为最高次项系数，开平方。符合所问。

12.

【原文】

今有勾股积三十步。只云勾股和一十七步。问：弦幂几何？

答曰：一百六十九步。

术曰：立天元一为弦幂，地元一为股，天、地配合求之。得二千八百

11. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the square of the *gu*.

Ans. 144 *bu*.

Process. Let the element *tian* be the square of the *gu* and the element *di* the *gou*. Using these elements we obtain 3600 for the negative *shi*, 169 for the positive *fang*, and 1 for the negative *yu*, a quadratic expression [1] whose root is the square of the *gu*.

[Notes]

[1] The expression in modern form is the equation: $-x^2 + 169x - 3600 = 0$. (C)

12. The product of the *gou* and the *gu* is 30 *bu*, and their sum 17 *bu*. Find the square of the *xian*.

Ans. 169 *bu*.

Process. Let the element *tian* be the square of the *xian* and the element *di* the *gu*. Using these elements we obtain 2873 for the positive *shi* and 17 for

七十三为正实，一十七为益方，开无隅平方。[1] 合问。

【注释】

[1] 开方式的现代形式为：$-17x+2873=0$。（陈）

【今译】

今有勾股形，面积为 30 步。只云勾股和 17 步。问：弦幂为多少？

答：169 步。

术：设天元一为弦幂，地元一为股，天元与地元相配合以求其解。得到 2873 为常数项，−17 为一次项系数，开无隅平方。符合所问。

the negative *fang*, a linear expression [1] whose root is the square of the *xian*.

[Notes]

[1] The expression in modern form is the equation: $-17x + 2873 = 0$. (C)

左右逢元[1] 二十一问

【注释】

[1] 此门的问题其实与前一门的属于同类，即求解高阶齐次方程的方法。朱世杰给此门选择的这样的标题无疑是引自《孟子》的"(取之)左右逢(其)源"。其音义均未变，仅是改写了最后一个字。(陈)

1.

【原文】

今有弦和较乘勾得六步。只云弦较较除股幂得四步。[1]问：勾、股几何？

答曰：勾三步，股四步。[2]

术曰：立天元一为股，地元一为较，天、地配合求之。得三千七十二为正实，七百八十四为益上廉，一百六十八为从下廉，五为益隅，三乘方开之。[3]合问。

【注释】

[1] 记勾、股、弦分别为a，b，c，此即：$[(a+b)-c]a=6$，$\dfrac{b^2}{c-(b-a)}=4$。(郭)

[2] 此下原脱"股四步"四字。(郭)

[3] 开方式的现代形式为：$-5x^4+168x^3-784x^2+3072=0$。(陈)

【今译】

今有弦与勾股和之差乘勾，得到6步。只云股幂除以弦与勾股差之差，得到4步。问：勾、股各为多少？

Zuo You Feng Yuan (Left, Right, You Meet Elements) [1]

21 Problems

[Notes]

[1] The problems under this head are in reality the same as those under the preceding head, namely the solution of simultaneous equations of higher degree. The author, in choosing the heading, no doubt is applying the quotation from Mencius, which does not differ in sound and differs in meaning only by changing the last character. Mencius, （取之）左右逢（其）源, go to the left or go to the right you meet springs. (C)

1. The product of the difference between the *he* and the *xian* by the *gou* is 6 *bu*; the quotient of the square of the *gu* and the difference between the *xian* and the *jiao* is 4 *bu*. [1] Find the *gou* and the *gu*.

Ans. *Gou*, 3 *bu*; *gu*, 4 *bu*.[2]

Process. Let the element *tian* be the *gu* and the element *di* the *jiao*. Using these elements we obtain 3072 for the positive *shi*, 784 for the negative upper *lian*, 168 for the positive lower *lian*, and 5 for the negative *yu*, an expression[3] of the fourth degree whose root is the required *gou*.

[Notes]

[1] Let the *gou* be a, the *gu* b, and the *xian* c. From the statement we have $[(a + b) - c]a = 6$, $\dfrac{b^2}{c - (b - a)} = 4$. (G)

[2] Following the answer, *gu*, 4 *bu* could be lost. (G)

[3] The expression in modern form is the equation: $-5x^4 + 168x^3 - 784x^2 + 3072 = 0$. (C)

答：勾 3 步，股 4 步。

术：设天元一为股，地元一为勾股差，天元与地元相配合以求其解。得到 3072 为常数项，−784 为二次项系数，168 为三次项系数，−5 为最高次项系数，开四次方。符合所问。

2.

【原文】

今有勾弦相乘，比直积多三步。只云股弦相乘，比弦幂少五步。[1] 问：勾、股几何？

答曰：勾三步，股四步。

术曰：立天元一为股，地元一为股弦较，天、地配合求之。得一十二为正实，三为益方，开无隅平方。[2] 合问。

【注释】

[1] 此即：$ac-ab=3, c^2-bc=5$。（郭）

[2] 开方式的现代形式为：$-3x+12=0$。（陈）

【今译】

今有勾弦相乘比以勾、股为边长的长方形面积多 3 步。只云股弦相乘比弦幂少 5 步。问：勾、股各为多少？

答：勾 3 步，股 4 步。

术：设天元一为股，地元一为股弦较，天元与地元相配合以求其解。得到 12 为常数项，−3 为一次项系数，开无隅平方。符合所问。

2. The product of the *gou* and the *xian* exceeds the *zhi ji* by 3 *bu*; the product of the *xian* and the *gu* is less than the square of the *xian* by 5 *bu*.[1] Find the *gou* and the *gu*.

Ans. *Gou*, 3 *bu*; *gu*, 4 *bu*.

Process. Let the element *tian* be the *gu* and the element *di* the difference between the *xian* and the *gu*. Using these elements we obtain 12 for the positive *shi* and 3 for the negative *fang*, a linear expression[2] whose root is the required *gu*.

[Notes]

[1] That is, $ac - ab = 3, c^2 - bc = 5$. (G)

[2] The expression in modern form is the equation: $-3x + 12 = 0$. (C)

3.

【原文】

今有直积加平，减二较，以长乘之，减积，以平除之，加二较，共得一十五步三分步之一。只云平幂减和，与二较等。[1] 问：长、平各几何？

答曰：平三步，长四步。

术曰：立天元一为长，地元一为平，天、地配合求之。得一千八百四十为益实，一千六十为从方，二百一十为从上廉，五十四为益下廉，九为益隅，三乘方开之，[2] 得长。合问。

【注释】

[1] 记平、长分别为a,b，此即：$\dfrac{[ab+a-2(b-a)]b-ab}{a}+2(b-a)=15\frac{1}{3}$；并且$a^2-(a+b)=2(b-a)$。（郭）

[2] 开方式的现代形式为：$-9x^4-54x^3+210x^2+1060x-1840=0$。（陈）

【今译】

今有长方形面积加阔，减2倍的长阔差，以长乘之，减长方形面积，以阔除之，加2倍的长阔差，共得$15\frac{1}{3}$步。只云：阔之幂减长阔和，与2倍的长阔差相等。问：长、阔各为多少？

答：阔3步，长为4步。

术：设天元一为长，地元一为阔，天元与地元相配合以求其解。得到−1840为常数项，1060为一次项系数，210为二次项系数，−54为三次项系数，−9为最高次项系数，开四次方，得到长。符合所问。

3. From the *zhi ji* increased by the width subtract twice the *jiao*; multiply the remainder by the length and subtract the *zhi ji*; divide this remainder by the width and increase the quotient by twice the *jiao*; the result is $15\frac{1}{3}$ *bu*. The square of the width less the *he* is equal to twice the *jiao*. [1] Find the length and the width.

Ans. Width, 3 *bu*; length, 4 *bu*.

Process. Let the element *tian* be the length and the element *di* the width. Using these elements we obtain 1840 for the negative *shi*, 1060 for the positive *fang*, 210 for the positive upper *lian*, 54 for the negative lower *lian*, and 9 for the negative *yu*, an expression[2] of the fourth degree whose root is the required length.

[Notes]

[1] Let the width be a, and the length b. From the statement we have $\frac{[ab + a - 2(b - a)]b - ab}{a} + 2(b - a) = 15\frac{1}{3}$; and $a^2 - (a + b) = 2(b - a)$. (G)

[2] The expression in modern form is the equation: $-9x^4 - 54x^3 + 210x^2 + 1060x - 1840 = 0$. (C)

4.

【原文】

今有直积加勾幂，减股幂，以平乘之，减直积，与平等。只云和减三较，以长除之，与较同。[1] 问：长、平各几何？

答曰：平三步，长四步。

术曰：立天元一为平，地元一为长，天、地配合求之。得三为正实，一十一为从方，八为从上廉，七为益下廉，一为正隅，三乘方开之，[2] 得平。合问。

【注释】

[1] 此即：$(ab+a^2-b^2)a-ab=a$；并且 $\frac{(a+b)-3(b-a)}{b}=b-a$。（郭）

[2] 开方式的现代形式为：$x^4-7x^3+8x^2+11x+3=0$。（陈）

【今译】

今有长方形面积加勾幂，减股幂，以阔乘之，减长方形面积，与阔相等。只云：长阔和减3倍的长阔差，以长除之，与长阔差相等。问：长、阔各为多少？

答：阔3步，长为4步。

术：设天元一为平，地元一为长，天元与地元相配合以求其解。得到3为常数项，11为一次项系数，8为二次项系数，−7为三次项系数，1为最高次项系数，开四次方，得到阔。符合所问。

5.

【原文】

今有直积加一平，减三长，余有三步。只云平幂与较等。[1] 问：积几何？

答曰：三十六步。

4. From the *zhi ji* increased by the square of the width subtract the square of the length; multiply the remainder by the width and subtract from the product the *zhi ji*; the result is equal to the width. The difference between the *he* and three times the *jiao* divided by the length is equal to the *jiao*. [1] Find the length and the width.

Ans. Width, 3 *bu*; length, 4 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 3 for the positive *shi*, 11 for the positive *fang*, 8 for the positive upper *lian*, 7 for the negative lower *lian*, and 1 for the positive *yu*, an expression [2] of the fourth degree whose root is the required width.

【Notes】

[1] That is, $(ab + a^2 - b^2)a - ab = a$; and $\frac{(a+b) - 3(b-a)}{b} = b - a$. (G)

[2] The expression in modern form is the equation: $x^4 - 7x^3 + 8x^2 + 11x + 3 = 0$. (C)

5. The *zhi ji* plus the width minus three times the length is 3 *bu*; the square of the width equals the *jiao*. [1] Find the *zhi ji*.

Ans. 36 *bu*.

Process. Let the element *tian* be the *zhi ji* and the element *di* the width. Using

术曰：立天元一为直积，地元一为平，天、地配合求之。得三十六为益实，三十七为从方，三十七为益廉，一为正隅，立方开之，[2]得积。合问。

【注释】

[1] 此即：$ab + a - 3b = 3$；并且 $a^2 = b - a$。（郭）

[2] 开方式的现代形式为：$x^3 - 37x^2 + 37x - 36 = 0$。（陈）

【今译】

今有长方形面积加阔，减3倍的长，余3步。只云：阔之幂与长阔差相等。问：长方形面积为多少？

答：36步。

术：设天元一为长方形面积，地元一为阔，天元与地元相配合以求其解。得到-36为常数项，37为一次项系数，-37为二次项系数，1为最高次项系数，开立方，得到长方形面积。符合所问。

6.

【原文】

今有直积加小和、小较，减大和、大较，余五十二步。只云大长加小和、小较，平方开之，不及大平二步。[1] 问：长、平各几何？

答曰：平六步，长一十二步。

术曰：立天元一为平，地元一为长，天、地配合求之。得九十六为正实，六十八为从方，一十四为益上廉，六为从下廉，一为益隅，三乘方开之，[2] 得。合问。

these elements we obtain 36 for the negative *shi*, 37 for the positive *fang*, 37 for the negative *lian*, and 1 for the positive *yu*, a cubic expression [2] whose root is the required *zhi ji*.

【Notes】

[1] That is, $ab + a - 3b = 3$; and $a^2 = b - a$. (G)

[2] The expression in modern form is the equation: $x^3 - 37x^2 + 37x - 36 = 0$. (C)

6. The *zhi ji* plus the *xiao he* and the *xiao jiao* minus the *da he* and the *da jiao* equals 52 *bu*. The square root of the sum of the great length, the *xiao he*, and the *xiao jiao* is less than the great width by 2 *bu*. [1] Find the length and the width.

Ans. Width, 6 *bu*; length, 12 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 96 for the positive *shi*, 68 for the positive *fang*, 14 for the negative upper *lian*, 6 for the positive lower *lian*, and 1 for the negative *yu*, an expression [2] of the fourth degree whose root is the

【注释】

［1］罗士琳认为，大长、大平、大和、大较、小长、小平、小和、小较分别为 a，b，$a+b$，$b-a$，$\frac{b}{a}$，$\frac{a}{b}$，$\frac{b}{a}+\frac{a}{b}$，$\frac{b}{a}-\frac{a}{b}$，此即：$ab+[(\frac{b}{a}+\frac{a}{b})+(\frac{b}{a}-\frac{a}{b})]-[(a+b)+(b-a)]=52$；并且 $a-\sqrt{b+[(\frac{b}{a}+\frac{a}{b})+(\frac{b}{a}-\frac{a}{b})]}=2$。（郭）

［2］开方式的现代形为：$-x^4+6x^3-14x^2+68x+96=0$。（陈）

【今译】

今有长方形面积加小和小较，减大和大较，余52步。只云：长加小和小较，开平方，比阔少2步。问：长方形的长、阔各为多少？

答：阔6步，长12步。

术：设天元一为长方形的阔，地元一为长，天元与地元相配合以求其解。得到96为常数项，68为一次项系数，−14为二次项系数，6为三次项系数，−1为最高次项系数，开四次方，得到阔。符合所问。

7.

【原文】

今有直田，长自乘，减和，余九步。只云平自乘，减较，余八步。[1] 问：长、平各几何？

答曰：平三步，长四步。

术曰：立天元一为平，地元一为长，天、地配合求之。得六十三为正实，一十八为益方，一十六为益上廉，二为从下廉，一为正隅，三乘方开之，[2] 得平。合问。

【注释】

［1］此即：$b^2-(a+b)=9$，$a^2-(b-a)=8$。（郭）

required width.

[Notes]

[1] In Luo Shilin's viewpoint, great length, great width, *da he*, *da jiao*, small length, small width, *xiao he*, and *xiao jiao* are a, b, $a+b$, $b-a$, $\frac{b}{a}$, $\frac{a}{b}$, $\frac{b}{a}+\frac{a}{b}$, and $\frac{b}{a}-\frac{a}{b}$. From the statement we have $ab+[(\frac{b}{a}+\frac{a}{b})+(\frac{b}{a}-\frac{a}{b})]-[(a+b)+(b-a)]=52$; and $a-\sqrt{b+[(\frac{b}{a}+\frac{a}{b})+(\frac{b}{a}-\frac{a}{b})]}=2$. (G)

[2] The expression in modern form is the equation: $-x^4+6x^3-14x^2+68x+96=0$. (C)

7. A piece of land is in the form of a rectangle. The square of its length minus the *he* is equal to 9 *bu*; the square of its width minus the *jiao* is equal to 8 *bu*. [1] Find the length and the width.

Ans. Width, 3 *bu*; length, 4 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 63 for the positive *shi*, 18 for the negative *fang*, 16 for the negative upper *lian*, 2 for the positive lower *lian*, and 1 for the positive *yu*, an expression [2] of the fourth degree whose root is the required width.

[2] 开方式的现代形式为：$x^4+2x^3-16x^2-18x+63=0$。（陈）

【今译】

今有直田，其长自乘，减去长阔和，余9步。只云：阔自乘，减去长阔差，余8步。问：直田的长、阔各为多少？

答：阔3步，长4步。

术：设天元一为直田的阔，地元一为长，天元与地元相配合以求其解。得到63为常数项，−18为一次项系数，−16为二次项系数，2为三次项系数，1为最高次项系数，开四次方，得到阔。符合所问。

8.

【原文】

今有直积，加一长、二平，共得二十二步。只云长幂加平，减较幂，余一十八步。[1] 问：长、平各几何？

答曰：平三步，长四步。

术曰：立天元一为平，地元一为长，天、地配合求之。得一十八为正实，二十七为益方，四为从廉，一为从隅，立方开之，[2] 得平。合问。

【注释】

[1] 此即：$ab+b+2a=22$，$b^2+a-(b-a)^2=18$。（郭）

[2] 开方式的现代形式为：$x^3+4x^2-27x+18=0$。（陈）

【今译】

今有长方形面积，加1倍的长，2倍的阔，共得22步。只云：长的幂加阔，减去长阔差之幂，余18步。问：长、阔各为多少？

答：阔3步，长4步。

术：设天元一为长方形的阔，地元一为长，天元与地元相配合以求其解。得到18为常数项，−27为一次项系数，4为二次项系数，1为最高次项系数，开立方，得到阔。符合所问。

【Notes】

[1] That is, $b^2 - (a + b) = 9$, $a^2 - (b - a) = 8$. (G)

[2] The expression in modern form is the equation: $x^4 + 2x^3 - 16x^2 - 18x + 63 = 0$. (C)

8. The sum of the *zhi ji*, the length, and twice the width is equal to 22 *bu*; the square of the length plus the width minus the square of the *jiao* is equal to 18 *bu*. [1] Find the length and the width.

Ans. Width, 3 *bu*; length, 4 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 18 for the positive *shi*, 27 for the negative *fang*, 4 for the positive *lian*, and 1 for the positive *yu*, a cubic expression [2] whose root is the required width.

【Notes】

[1] That is, $ab + b + 2a = 22$, $b^2 + a - (b - a)^2 = 18$. (G)

[2] The expression in modern form is the equation: $x^3 + 4x^2 - 27x + 18 = 0$. (C)

9.

【原文】

今有直积，加二平，减三较，余一十五步。只云长取强半，平取少半，与和七分之四等。[1] 问：长、平各几何？

答曰：平三步，长四步。

术曰：立天元一为平，地元一为长，天、地配合求之。得四十五为益实，三为从方，四为从隅，平方开之，[2] 得平。合问。

【注释】

[1] 此即：$ab + 2a - 3(b - a) = 15$，$\frac{3}{4}b + \frac{1}{3}a = \frac{4}{7}(a + b)$。（郭）

[2] 开方式的现代形式为：$4x^2 + 3x - 45 = 0$。（陈）

【今译】

今有长方形面积，加2倍的阔，减去3倍的长阔差，余15步。只云：取长的$\frac{3}{4}$，阔的$\frac{1}{4}$，其和与长阔和的$\frac{4}{7}$相等。问：长、阔各为多少？

答：阔3步，长4步。

术：设天元一为长方形的阔，地元一为长，天元与地元相配合以求其解。得到−45为常数项，3为一次项系数，4为最高次项系数，开平方，得到阔。符合所问。

10.

【原文】

今有直积，以长乘之，用平除之，所得减积，如长而一，得七步。只云较幂加长，与平幂同。[1] 问：长、平各几何？

答曰：平八步，长一十五步。

9. The *zhi ji* plus twice the width minus three times the *jiao* is equal to 15 *bu*. If we take the strong half of the length and the less half of the width their sum is equal to four-sevenths of the *he*. [1] Find the length and the width.

Ans. Width, 3 *bu*; length, 4 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 45 for the negative *shi*, 3 for the positive *fang*, and 4 for the positive *yu*, a quadratic expression [2] whose root is the required width.

[Notes]

[1] That is, $ab + 2a - 3(b - a) = 15, \frac{3}{4}b + \frac{1}{3}a = \frac{4}{7}(a + b)$. (G)

[2] The expression in modern form is the equation: $4x^2 + 3x - 45 = 0$. (C)

10. Multiply the *zhi ji* by the length and divide by the width; from the quotient subtract the *zhi ji* and divide the remainder by the length; the quotient is 7 *bu*. The square of the *jiao* increased by the length is equal to the square of the width. [1] Find the length and the width.

Ans. Width, 8 *bu*; length, 15 *bu*.

术曰：立天元一为长，地元一为平，天、地配合求之。得一十五为益实，一为从方，上实，下法而一，[2] 得长。合问。

【注释】

[1] 此即：$\dfrac{\dfrac{(ab)b}{a}-ab}{b}=7$，$(b-a)^2+b=a^2$。（郭）

[2] 开方式的现代形式为：$x-15=0$。（陈）

【今译】

今有长方形面积，乘以长，除以平，所得减去面积，除以长，得 7 步。只云：长阔差之幂加长，与阔之幂相等。问：长、阔各为多少？

答：阔 8 步，长 15 步。

术：设天元一为长方形的阔，地元一为长，天元与地元相配合以求其解。得到 -15 为常数项，1 为一次项系数，15 作为被除数，1 作为除数，除之，得到长。符合所问。

11.

【原文】

今有直田，平幂减一和、六较，余与长等。只云较幂加一平，减四较，亦与长等。[1] 问：长、平各几何？

答曰：平八步，长一十三步。

术曰：立天元一为平，地元一为长，天、地配合求之。得一百二十为益实，三十一为从方，六为从廉，一为益隅，立方开之，[2] 得平。合问。

【注释】

[1] 此即：$a^2-[(a+b)+6(b-a)]=b$，$(b-a)^2+a-4(b-a)=b$。（郭）

Process. Let the element *tian* be the length and the element *di* the width. Using these elements we obtain 15 for the negative *shi* and 1 for the positive *fang*, a linear expression [2] whose root, by division, is the required length.

[Notes]

[1] That is, $\frac{\frac{(ab)b}{a} - ab}{b} = 7$, $(b - a)^2 + b = a^2$. (G)

[2] The expression in modern form is the equation: $x - 15 = 0$. (C)

11. In a right triangle from the square of the width subtract the sum of the *he* and six times the *jiao*; the remainder is equal to the length. The square of the *jiao* plus the width less four times the *jiao* is also equal to the length. [1] Find the length and the width.

Ans. Width, 8 *bu*; length, 13 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 120 for the negative *shi*, 31 for the positive *fang*, 6 for the positive *lian*, and 1 for the negative *yu*, a cubic expression [2] whose root is the required width.

[Notes]

[1] That is, $a^2 - [(a + b) + 6(b - a)] = b$, $(b - a)^2 + a - 4(b - a) = b$. (G)

［2］开方式的现代形式为：$-x^3+6x^2+31x-120=0$。（陈）

【今译】

今有直田，其阔幂减1倍的阔与6倍的长阔差，余数与长相等。只云：长阔差幂加阔，减去4倍的长阔差，亦与长相等。问：长、阔各为多少？

答：阔8步，长13步。

术：设天元一为直田的阔，地元一为长，天元与地元相配合以求其解。得到 -120 为常数项，31为一次项系数，6为二次项系数，-1 为最高次项系数，开立方，得到阔。符合所问。

12.

【原文】

今有直积，加和幂，减较幂，以平除之，与积等。只云长幂加二较，减二差幂，亦与积等。[1] 问：长、平各几何？

答曰：平五步，长一十二步。

术曰：立天元一为长，地元一为平，天、地配合求之。得六十为益实，一十七为从方，一为益隅，平方开之，[2] 得长。合问。

【注释】

［1］此即：$\frac{ab+(a+b)^2-(b-a)^2}{a}=ab$，$b^2+2(b-a)-2(b-a)^2=ab$。（郭）

［2］开方式的现代形式为：$-x^2+17x-60=0$。（陈）

【今译】

今有长方形面积，加长阔和之幂，减去长阔差之幂，以阔除之，与长方

[2] The expression in modern form is the equation: $-x^3 + 6x^2 + 31x - 120 = 0$. (C)

12. Add to the *zhi ji* the square of the *he* and subtract the square of the *jiao*, then divide the result by the width; the quotient is equal to the *zhi ji*. Add to the square of the length twice the *jiao* and subtract twice the square of the *jiao*; the result is equal to the *zhi ji*. [1] Find the length and the width.

Ans. Width, 5 *bu*; length, 12 *bu*.

Process. Let the element *tian* be the length and the element *di* the width. Using these elements we obtain 60 for the negative *shi*, 17 for the positive *fang*, and 1 for the negative *yu*, a quadratic expression [2] whose root is the required length.

[Notes]

[1] That is, $\frac{ab + (a+b)^2 - (b-a)^2}{a} = ab$, $b^2 + 2(b-a) - 2(b-a)^2 = ab$. (G)

[2] The expression in modern form is the equation: $-x^2 + 17x - 60 = 0$. (C)

形面积相等。只云：长幂加2倍的长阔差，减2倍的长阔差幂，亦与长方形面积相等。问：长、阔各为多少？

答：阔5步，长12步。

术：设天元一为长方形的长，地元一为平，天元与地元相配合以求其解。得到 −60 为常数项，17 为一次项系数，−1 为最高次项系数，开平方，得到长。符合所问。

13.

【原文】

今有直积，减小平，加大较、小和，多积五步。只云二大和减小长、大平，少积五步。[1] 问：长、平各几何？

答曰：平三步，长六步。

术曰：立天元一为平，地元一为长，天、地配合求之。得一十五为益实，二为从方，一为从隅，平方开之，[2] 得平。合问。

【注释】

[1] 据《算学启蒙·总括》，大和、大较、小长、小平、小和分别为 $a+b$，$b-a$，$\frac{b}{a}$，$\frac{a}{b}$，$\frac{b}{a}+\frac{a}{b}$，此即：$\{ab-\frac{a}{b}+[(b-a)+(\frac{b}{a}+\frac{a}{b})]\}-ab=5$，$ab-[2(a+b)-(\frac{b}{a}+a)]=5$。（郭）

[2] 开方式的现代形式为：$x^2+2x-15=0$。（陈）

【今译】

今有长方形面积，减小平，加大较、小和，比长方形面积多5步。只云：2倍的大和减去小长与大平，比长方形面积少5步。问：长、阔各为多少？

13. The *zhi ji* minus the sum of the small width, the great *jiao*, and the small *he* exceeds the *zhi ji* by 5 *bu*. Twice the great he minus the sum of the small length and the great width is less than the *zhi ji* by 5 *bu*.[1] Find the length and the width.

Ans. Width, 3 *bu*; length, 6 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 15 for the negative *shi*, 2 for the positive *fang*, and 1 for the positive *yu*, a quadratic expression [2] whose root is the required width.

[Notes]

[1] According to the *zongkuo* of the *Introduction to Mathematical Studies*, the great *he* should be $a + b$, the great *jiao* $b - a$, the small length $\frac{b}{a}$, the small width $\frac{a}{b}$, and the small *he* $\frac{b}{a} + \frac{a}{b}$. From the statement we have $\{ab - \frac{a}{b} + [(b - a) + (\frac{b}{a} + \frac{a}{b})]\} - ab = 5, ab - [2(a + b) - (\frac{b}{a} + a)] = 5$. (G)

[2] The expression in modern form is the equation: $x^2 + 2x - 15 = 0$. (C)

答：阔3步，长6步。

术：设天元一为长方形的平，地元一为长，天元与地元相配合以求其解。得到-15为常数项，2为一次项系数，1为最高次项系数，开平方，得到平。符合所问。

14.

【原文】

今有直积，平方开之，减平，余有三步。只云长以平方开之，不及较三步。[1] 问：长、平各几何？

答曰：平九步，长一十六步。

术曰：立天元一为平，地元一为长，天、地配合求之。得八十一为正实，四十五为从方，三为从廉，一为益隅，立方开之，[2] 得平。合问。

【注释】

[1] 此即：$\sqrt{ab}-a=3$，$(b-a)-\sqrt{b}=3$。（郭）

[2] 开方式的现代形式为：$-x^3+3x^2+45x+81=0$。（陈）

【今译】

今有长方形面积，将其开平方，减去阔，余有3步。只云：将长开平方，比长阔差少3步。问：长、阔各为多少？

答：阔9步，长16步。

术：设天元一为直田的阔，地元一为长，天元与地元相配合以求其解。得到81为常数项，45为一次项系数，3为二次项系数，-1为最高次项系数，开立方，得到阔。符合所问。

14. The square root of the *zhi ji* minus the width is 3 *bu*; the square root of the length is less than the *jiao* by 3 *bu*. [1] Find the length and the width.

Ans. Width, 9 *bu*; length, 16 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 81 for the positive *shi*, 45 for the positive *fang*, 3 for the positive *lian*, and 1 for the negative *yu*, a cubic expression [2] whose root is the required width.

[Notes]

[1] That is, $\sqrt{ab} - a = 3$, $(b - a) - \sqrt{b} = 3$. (G)

[2] The expression in modern form is the equation: $-x^3 + 3x^2 + 45x + 81 = 0$. (C)

15.

【原文】

今有直积及和，各以平方开之，所得相并，减平，余八步。只云：长以平方开之，少如和开方数一步。[1] 问：长、平各几何？

答曰：平九步，长一十六步。

术曰：立天元一为平，地元一为和开方数，天、地配合求之。得二百二十五为正实，一百九十六为益方，二十六为益上廉，四为益下廉，一为正隅，三乘方开之，[2] 得平。合问。

【注释】

[1] 此即：$[\sqrt{ab}+\sqrt{(a+b)}]-a=8$，$\sqrt{a+b}-\sqrt{b}=1$。（郭）

[2] 开方式的现代形式为：$x^4-4x^3-26x^2-196x+225=0$。（陈）

【今译】

今有长方形面积及长阔和，各自开平方，其得数相加，减去阔，余 8 步。只云：将长开平方，比长阔和的开平方得数少 1 步。问：直田的长、阔各为多少？

答：阔 9 步，长 16 步。

术：设天元一为长方形的阔，地元一为长阔和的开平方得数，天元与地元相配合以求其解。得到 225 为常数项，−196 为一次项系数，−26 为二次项系数，−4 为三次项系数，1 为最高次项系数，开四次方，得到阔。符合所问。

16.

【原文】

今有直积加平，与二和、一较等。只云长幂减较幂，亦与二和、一较等。[1] 问：四事各几何？

15. The sum of the square roots of the *zhi ji* and the *he* minus the width is 8 *bu*; the square root of the length is less than the square root of the *he* by 1 *bu*. [1] Find the length and the width.

Ans. Width, 9 *bu*; length, 16 *bu*.

Process. Let the element *tian* be the width and the element *di* the *he*. Using these elements we obtain 225 for the positive *shi*, 196 for the negative *fang*, 26 for the negative upper *lian*, 4 for the negative lower *lian*, and 1 for the positive *yu*, an expression [2] of the fourth degree whose root is the required width.

[Notes]

[1] That is, $[\sqrt{ab} + \sqrt{(a+b)}] - a = 8$, $\sqrt{a+b} - \sqrt{b} = 1$. (G)

[2] The expression in modern form is the equation: $x^4 - 4x^3 - 26x^2 - 196x + 225 = 0$. (C)

16. The sum of the *zhi ji* and the width is equal to the sum of the *jiao* and twice the *he*; the difference between the squares of the length and the *jiao* is also equal to the sum of the *jiao* and twice the *he*. [1] What are the four matters [length, width, *he*, and *jiao*]?

答曰：平三步，长四步。

和七步，较一步。[2]

术曰：立天元一为平，地元一为长，天、地配合求之。得三为正实，二为从方，一为益隅，平方开之，[3] 得平。 又：立天元一为长，地元一为平。求得一十二为益实，三为从方，开无隅平方而一，[4] 得长。又：立天元一为和，地元一为平。求得七为正实，六为从方，一为益隅，平方开之，[5] 得和。 又：立天元一为较，地元一为长。求得三为正实，三为益方，上实，下法而一，[6] 即较。合问。

【注释】

[1] 此即：$ab + a = 2(a + b) + (b - a)$，$b^2 - (b - a)^2 = 2(a + b) + (b - a)$。（郭）

[2] 原本脱“和七步，较一步”六字，据设问与术文校补。（郭）

[3] 开方式的现代形式为：$-x^2 + 2x + 3 = 0$。（陈）

[4] 开方式的现代形式为：$3x - 12 = 0$。（陈）

[5] 开方式的现代形式为：$-x^2 + 6x + 7 = 0$。（陈）

[6] 开方式的现代形式为：$-3x + 3 = 0$。（陈）

【今译】

今有长方形面积加阔，等于2倍的长阔和与1倍的长阔差之和。只云：长幂减长阔差幂，也等于2倍的长阔和与1倍的长阔差之和。问：长、阔、长阔和、长阔差四者各为多少？

答：阔3步，长4步。

长阔和为7步，长阔差为1步。

术：设天元一为长方形的阔，地元一为长，天元与地元相配合以求其解。得到3为常数项，2为一次项系数，−1为最高次项系数，开平方，得到阔。 又：设天元一为长方形的长，地元一为阔，天元与地元相配合以求其解。得到−12为常数项，3为一次项系数，开无隅平方，得到长。 又：设天元一为长方形的长阔和，地元一为阔，天元与地元相配合以求其解。得到7为常数项，6为一次项系数，−1为最高次项系数，开平方，得到长阔和。 又：设天元一为长方形的长阔差，地元一为长，天元与地元相配合以求其解。得到3为常数项，−3为一次项系数，常数项3作为实，一次项系数3作为法，除之，得到长阔差。符合所问。

Ans. Width, 3 *bu*; length, 4 *bu*.

He, 7 *bu*; *jiao* 1 *bu*. [2]

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 3 for the positive *shi*, 2 for the positive *fang*, and 1 for the negative *yu*, a quadratic expression [3] whose root is the required width. Again let the element *tian* be the length and the element *di* the width. Using these elements we obtain 12 for the negative *shi* and 3 for the positive *fang*, a linear expression [4] whose root is the required length. Again let the element *tian* be the *he* and the element *di* the width. Using these elements we obtain 7 for the positive *shi*, 6 for the positive *fang*, and 1 for the negative *yu*, a quadratic expression [5] whose root is the required *he*. Again let the element *tian* be the *jiao* and the element *di* the length. Using these elements we obtain 3 for the positive *shi* and 3 for the negative *fang*, the linear expression [6] whose root, obtained by division, is the required *jiao*.

[Notes]

[1] That is, $ab + a = 2(a + b) + (b - a)$, $b^2 - (b - a)^2 = 2(a + b) + (b - a)$. (G)

[2] The original text lost the six characters *he qi bu, jiao yi bu*. According to the statement and the question, we add them. (G)

[3] The expression in modern form is the equation: $-x^2 + 2x + 3 = 0$. (C)

[4] The expression in modern form is the equation: $3x - 12 = 0$. (C)

[5] The expression in modern form is the equation: $-x^2 + 6x + 7 = 0$. (C)

[6] The expression in modern form is the equation: $-3x + 3 = 0$. (C)

17.

【原文】

今有平乘直积，与一长、五和等。只云长幂加较幂，与一长、三和等。[1] 问：长、平各几何？

答曰：平三步，长五步。

术曰：立天元一为平，地元一为长，天、地配合求之。得一百八为益实，三十六为从方，三十六为从上廉，一十二为益二廉，三为益下廉，一为正隅，四乘方开之，[2] 得平。 又：立天元一为长，地元一为平。求得五为正实，七十六为益方，五十五为从上廉，二为从二廉，八为从下廉，二为益隅，四乘方开之，[3] 得长。合问。

【注释】

[1] 此即：$(ab)a = b + 5(a + b)$；并且 $b^2 + (b - a)^2 = b + 3(a + b)$。（郭）

[2] 开方式的现代形式为：$x^5 - 3x^4 - 12x^3 + 36x^2 + 36x - 108 = 0$。（陈）

[3] 开方式的现代形式为：$-2x^5 + 8x^4 + 2x^3 + 55x^2 - 76x + 5 = 0$。（陈）

【今译】

今有阔乘长方形面积，等于长与5倍的长阔和之和。只云：长幂加长阔差幂，等于长与3倍的长阔和之和。问：长、阔各为多少？

答：阔3步，长为5步。

术：设天元一为阔，地元一为长，天元与地元相配合以求其解。得到−108为常数项，36为一次项系数，36为二次项系数，−12为三次项系数，−3为四次项系数，1为最高次项系数，开五次方，得到阔。 又：设天元一为长，地元一为阔，天元与地元相配合以求其解。得到5为常数项，−76为一次项系数，55为二次项系数，2为三次项系数，8为四次项系数，−2为最高次项系数，开五次方，得到长。符合所问。

17. The product of the width and the *zhi ji* is equal to the sum of the length and five times the *he*; the sum of the squares of the length and the *jiao* is equal to the sum of the length and three times the *he*.[1] Find the length and the width.

Ans. Width, 3 *bu*; length, 5 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 108 for the negative *shi*, 36 for the positive *fang*, 36 for the positive upper *lian*, 12 for the negative second *lian*, 3 for the negative lower *lian*, and 1 for the positive *yu*, an expression [2] of the fifth degree whose root is the required width. Again let the element *tian* be the length and the element *di* the width. Using these elements we obtain 5 for the positive *shi*, 76 for the negative *fang*, 55 for the positive upper *lian*, 2 for the positive second *lian*, 8 for the positive lower *lian*, and 2 for the negative *yu*, an expression [3] of the fifth degree whose root is the required length.

[Notes]

[1] That is, $(ab)a = b + 5(a+b)$; and $b^2 + (b-a)^2 = b + 3(a+b)$. (G)

[2] The expression in modern form is the equation: $x^5 - 3x^4 - 12x^3 + 36x^2 + 36x - 108 = 0$. (C)

[3] The expression in modern form is the equation: $-2x^5 + 8x^4 + 2x^3 + 55x^2 - 76x + 5 = 0$. (C)

18.

【原文】

今有直积，三乘方开之，得数以平除之，不及平三步。只云长以平方开之，多于平二分之一。[1] 问：长、平各几何？

答曰：平四步，长六十四步。

术曰：立天元一为平，地元一为长，天、地配合求之。得四为正实，八十一为益方，一百八为从上廉，五十四为益二廉，一十二为从下廉，一为益隅，四乘方开之，[2] 得平。合问。

【注释】

[1] 此即：$a-\frac{\sqrt[4]{ab}}{a}=3$；并且 $\sqrt{b}-a=\frac{1}{2}\sqrt{b}$。（郭）

[2] 开方式的现代形式为：$-x^5+12x^4-54x^3+108x^2-81x+4=0$。（陈）

【今译】

今有长方形面积，开四次方，其得数以阔除之，比阔少3步。只云：长开平方，比阔多长开平方得数的$\frac{1}{2}$。问：长、阔各为多少？

答：阔4步，长为64步。

术：设天元一为阔，地元一为长，天元与地元相配合以求其解。得到4为常数项，−81为一次项系数，108为二次项系数，−54为三次项系数，12为四次项系数，−1为最高次项系数，开五次方，得到阔。符合所问。

19.

【原文】

今有直积，三乘方开之，以平而一，所得，少平三步。只云长以平方开

18. Divide the fourth root of the *zhi ji* by the width; the quotient is less than the width by 3 *bu*; the square root of the length exceeds the width by half of itself.[1] Find the length and the width.

Ans. Width, 4 *bu*; length, 64 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 4 for the positive *shi*, 81 for the negative *fang*, 108 for the positive upper *lian*, 54 for the negative second *lian*, 12 for the positive lower *lian*, and 1 for the negative *yu*, an expression [2] of the fifth degree whose root is the required width.

[Notes]

[1] That is, $a - \frac{\sqrt[4]{ab}}{a} = 3$; and $\sqrt{b} - a = \frac{1}{2}\sqrt{b}$. (G)

[2] The expression in modern form is the equation: $-x^5 + 12x^4 - 54x^3 + 108x^2 - 81x + 4 = 0$. (C)

19. Divide the fourth root of the *zhi ji* by the width; the quotient is less than the width by 3 *bu*; the square root of the length is less than the length by seven-eighths [of itself].[1] Find the length and the width.

之，不及长八分之七。[1] 问：长、平各几何？

答曰：平四步，长六十四步。

术曰：立天元一为平，地元一为长，天、地配合求之。得六十四为益实，八十一为从二廉，一百八为益三廉，五十四为从四廉，一十二为益下廉，一为正隅，六乘方开之，[2] 得平。合问。

【注释】

[1] “以平而一”之“以”，罗士琳改作“如”，无必要。此即：$a-\frac{\sqrt[4]{ab}}{a}=3$；并且 $\sqrt{b}=\left(1-\frac{7}{8}\right)b$。（郭）

[2] 开方式的现代形式为：$x^7-12x^6+54x^5-108x^4+81x^3-64=0$。（陈）

【今译】

今有长方形面积，开四次方，其得数以阔除之，比阔少 3 步。只云：长开平方，不及长 $\frac{7}{8}$。问：长、阔各为多少？

答：阔 4 步，长为 64 步。

术：设天元一为阔，地元一为长，天元与地元相配合以求其解。得到 −64 为常数项，81 为三次项系数，−108 为四次项系数，54 为五次项系数，−12 为六次项系数，1 为最高次项系数，开七次方，得到阔。符合所问。

20.

【原文】

今有直积，减小平，加小较，以大平乘之，如大长而一。得数，减小平，余有九步。只云平幂加大较，如大长而一，加大长，得八步。[1] 问：长、平各几何？

Ans. Width, 4 *bu*; length, 64 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 64 for the negative *shi*, 81 for the positive second *lian*, 108 for the negative third *lian*, 54 for the positive fourth *lian*, 12 for the negative lower *lian*, and 1 for the positive *yu*, an expression [2] of the seventh degree whose root is the required width.

[Notes]

[1] It is unnecessary that Luo Shilin changed the character *yi* of *yi ping er yi* into *ru*. That is, $a - \frac{\sqrt[4]{ab}}{a} = 3$; and $\sqrt{b} = (1 - \frac{7}{8})\,b$. (G)

[2] The expression in modern form is the equation: $x^7 - 12x^6 + 54x^5 - 108x^4 + 81x^3 - 64 = 0$. (C)

20. Multiply the *zhi ji* less the small width plus the small *jiao* by the great width, divide the product by the great length and from the quotient subtract the small width; the remainder is equal to 9 *bu*. Divide the square of the width plus the great *jiao* by the great length and add to the quotient the great width; the result is equal to 8 *bu*. [1] Find the length and the width.

答曰：平三步，长六步。

术曰：立天元一为平，地元一为长，天、地配合求之。得五万六百八十八为益实，一千七百二十八为从方，一万七千一百五十二为从上廉，一千七百七十六为益二廉，一千二百八为益三廉，四百九十一为从四廉，一百七十为益五廉，五十三为益六廉，二十七为从七廉，二为从八廉，一为益隅，九乘方开之，[2] 得平。合问。

【注释】

[1] 此即：$\dfrac{\left[ab-\frac{a}{b}+\left(\frac{b}{a}-\frac{a}{b}\right)\right]a}{b}-\dfrac{a}{b}=9$；并且$\dfrac{a^2+(b-a)}{b}+b=8$。（郭）

[2] 开方式的现代形式为：

$$-x^{10}+2x^9+27x^8-53x^7-170x^6+491x^5-1208x^4-1776x^3+17152x^2+1728x-50688=0$$。（陈）

【今译】

今有长方形面积，减小平，加小较，以阔乘之，除以长。其得数，减去小平，余有9步。只云：阔之幂加大较，除以长，加长，得8步。问：长、阔各为多少？

答：阔3步，长为6步。

术：设天元一为阔，地元一为长，天元与地元相配合以求其解。得到−50688为常数项，1728为一次项系数，17152为二次项系数，−1776为三次项系数，−1208为四次项系数，491为五次项系数，−170为六次项系数，−53为七次项系数，27为八次项系数，2为九次项系数，−1为最高次项系数，开十次方，得到阔。符合所问。

Ans. Width, 3 *bu*; length, 6 *bu*.

Process. Let the element *tian* be the width and the element *di* the length. Using these elements we obtain 50688 for the negative *shi*, 1728 for the positive *fang*, 17152 for the positive upper *lian*, 1776 for the negative second *lian*, 1208 for the negative third *lian*, 491 for the positive fourth *lian*, 170 for the negative fifth *lian*, 53 for the negative sixth *lian*, 27 for the positive seventh *lian*, 2 for the positive eighth *lian*, and 1 for the negative *yu*, an expression [2] of the tenth degree whose root is the required width.

[Notes]

[1] That is, $\frac{[ab - \frac{a}{b} + (\frac{b}{a} - \frac{a}{b})]a}{b} - \frac{a}{b} = 9$; and $\frac{a^2 + (b - a)}{b} + b = 8$. (G)

[2] The expression in modern form is the equation: $-x^{10} + 2x^9 + 27x^8 - 53x^7 - 170x^6 + 491x^5 - 1208x^4 - 1776x^3 + 17152x^2 + 1728x - 50688 = 0$. (C)

21.

【原文】

今有勾弦相乘，加勾股较，平方开之，与股适等。只云股弦相乘，减弦和和，立方开之，与勾弦较同。[1] 问：勾、股[2] 各几何？

答曰：勾三步，股四步。

术曰：立天元一为勾，地元一为股弦较，天、地配合求之。得六为益实，一十四为从方，一百一十为从上廉，六百二十为益二廉，一千五百二十为从三廉，二千四百四十六为益四廉，二千七百四十七为从五廉，一千九百三十二为益六廉，六百七十一为从七廉，二十二为从八廉，六十为益九廉，八为从隅，十乘方开之，[3] 得勾三步。合问。

【注释】

[1] 此即：$\sqrt{ac+(b-a)}=b$；并且 $\sqrt[3]{bc-[(a+b)+c]}=c-a$。（郭）

[2] “股”，原文讹作“弦”，依戴煦校正。（郭）

[3] 开方式的现代形式为：

$8x^{11}-60x^{10}+22x^9+671x^8-1932x^7+2747x^6-2446x^5+1520x^4-620x^3+110x^2+14x-6=0$。（陈）罗士琳认为此方程并非完全正确。（郭）

【今译】

今有勾弦相乘，加勾股较，开平方，其得数与股恰好相等。只云：股弦相乘，减去弦和和，开立方，其得数与勾弦较相等。问：勾、股各为多少？

答：勾 3 步，股 4 步。

术：设天元一为勾，地元一为股弦较，天元与地元相配合以求其解。得到 −6 为常数项，14 为一次项系数，110 为二次项系数，−620 为三次项系数，1520 为四次项系数，−2446 为五次项系数，2747 为六次项系数，−1932 为七次项系数，671 为八次项系数，22 为九次项系数，−60 为十次项系数，8 为最高次项系数，开十一次方，得到勾。符合所问。

21. The square root of the sum of the *gou* times the *xian* plus the *jiao* equals the *gu*; the cube of the difference between the *xian* times the *gu* and the sum of the *xian* and the *he* equals the *xian* less the *gou*.[1] Find the *gou* and the *gu*[2].

Ans. *Gou*, 3 *bu*; *gu*, 4 *bu*.

Process. Let the element *tian* be the *gou* and the element *di* the *xian* less the *gu*. Using these elements we obtain 6 for the negative *shi*, 14 for the positive *fang*, 110 for the positive upper *lian*, 620 for the negative second *lian*, 1520 for the positive third *lian*, 2446 for the negative fourth *lian*, 2747 for the positive fifth *lian*, 1932 for the negative sixth *lian*, 671 for the positive seventh *lian*, 22 for the positive eighth *lian*, 60 for the negative ninth *lian*, and 8 for the positive *yu*, an expression [3] of the eleventh degree whose root, 3 *bu*, is the required *gou*.

[Notes]

[1] That is, $\sqrt{ac + (b - a)} = b$; and $\sqrt[3]{bc - [(a + b) + c]} = c - a$. (G)

[2] The *gu* was mistaken as *xian*. According to Dai Xu's research, we revise it. (G)

[3] The expression in modern form is the equation:

$8x^{11} - 60x^{10} + 22x^9 + 671x^8 - 1932x^7 + 2747x^6 - 2446x^5 + 1520x^4 - 620x^3 + 110x^2 + 14x - 6 = 0$. (C) Luo Shilin considered that the equation was not completely right. (G)

三才变通 十一问

1.

【原文】

今有直积，减弦较和，加股弦较幂，与弦较较幂等。只云勾幂减三相和，与弦较和同。[1] 问：弦几何？

答曰：一十七步。

术曰：立天元一为勾，地元一为股，人元一为弦，三才相配求之。得三十四为正实，一十九为益方，一为正隅，平方开之，[2] 得弦。合问。

【注释】

[1] 此即：$ab-[(b-a)+c]+(c-b)^2=[c-(b-a)]^2$，$a^2-(a+b+c)=c+(b-a)$。（郭）

[2] 开方式的现代形式为：$z^2-19z+34=0$。（陈）

【今译】

今有长方形面积，减弦较和，加股弦较幂，与弦较较幂相等。只云：勾幂减勾、股、弦三者之和，与弦较和相等。问：弦为多少？

答：17 步。

术：设天元一为勾，地元一为股，人元一为弦，天元、地元与人元三者相配合以求其解。得到 34 为常数项，−19 为一次项系数，1 为最高次项系数，开平方，得到弦。符合所问。

2.

【原文】

今有直积，加黄方幂，开方除之，与倍之勾弦较等。只云：弦较较幂减勾股和，开方除之，与股少股弦较同。[1] 问：三相和几何？

San Cai Bian Tong (Changing of the Three Talents or Expressions in Three Unknown Quantities)

11 Problems

1. The *zhi ji* minus the sum of the *xian* and the *jiao* plus the square of the difference between the *xian* and the *gu* is equal to the square of the difference between the *xian* and the *jiao*; the square of the *gou* minus the sum of the three [*gou*, *gu* and *xian*] is equal to the sum of the *xian* and the *jiao*. [1] Find the *xian*.

Ans. *Xian*, 17 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the *xian*. Using these three talents [elements] we obtain 34 for the positive *shi*, 19 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression [2] whose root is the required *xian*.

[Notes]

[1] That is, $ab - [(b-a)+c] + (c-b)^2 = [c-(b-a)]^2$, $a^2 - (a+b+c) = c + (b-a)$. (G)

[2] The expression in modern form is the equation: $z^2 - 19z + 34 = 0$. (C)

2. The square root of the sum of the *zhi ji* and the square of the *huang fang* is equal to twice the difference between the *xian* and the *gou*; the square root of the difference between the *xian* less the *jiao* squared and the sum of the *gu* and the *gou* is equal to the *gu* less the difference between the *xian* and the

答曰：一十二步。

术曰：立天元一为勾，地元一为股，人元一为三相和，三才相配求之。得八百六十四为益实，四千一百四为益方，四万二千二百二十八为从上廉，五万三千九百九十八为益下廉，四千二百九为从隅，三乘方开之。[2] 合问。

【注释】

[1] 此即：$\sqrt{ab+[(a+b)-c]^2}=2(c-a)$，

$\sqrt{[c-(b-a)]^2-(a+b)}=b-(c-b)$。（郭）

[2] 开方式的现代形式为：$4209z^4-53998z^3+42228z^2-4104z-864=0$。（陈）

【今译】

今有长方形面积加黄方幂，开平方，其得数与2倍的勾弦较相等。只云：弦较较幂减去勾股和，开平方，其得数与股减股弦较相等。问：勾、股、弦三者之和为多少？

答：12步。

术：设天元一为勾，地元一为股，人元一为勾、股、弦三者之和，天元、地元与人元三者相配合以求其解。得到−864为常数项，−4104为一次项系数，42228为二次项系数，−53998为三次项系数，4209为最高次项系数，开四次方，即得。符合所问。

3.

【原文】

今有平乘积，如长而一，所得，减一平、三较，余与平等。只云长乘和，减平，与二积、一较同。[1] 问：和幂、弦幂、较幂，带一积、一长、一

gu. [1] Find the sum of the three (sides).

Ans. 12 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the sum of the three. Using these three talents we obtain 864 for the negative *shi*, 4104 for the negative *fang*, 42228 for the positive upper *lian*, 53998 for the negative lower *lian*, and 4209 for the positive *yu*, an expression[2] of the fourth degree whose root is the required sum.

[Notes]

[1] That is, $\sqrt{ab + [(a+b)-c]^2} = 2(c-a)$,

$\sqrt{[c-(b-a)]^2 - (a+b)} = b - (c-b)$. (G)

[2] The expression in modern form is the equation: $4209z^4 - 53998z^3 + 42228z^2 - 4104z - 864 = 0$. (C)

3. Multiply the width by the *zhi ji* and divide the product by the length; subtract the sum of the width and three times the *jiao*; the remainder is equal to the width. Multiply the *he* by the length and subtract the width; the remainder is equal to twice the *zhi ji* plus the *jiao*. [1] Find the sum of the six matters: the

平六事连环得几何？[2]

答曰：九十四步。

术曰：立天元一为勾，地元一为股，人元一为开数，三才相配求之。得一百八十八为正实，九十六为益方，一为正隅，平方开之。[3]合问。

【注释】

[1] 此即：$\frac{a(ab)}{b}-[a+3(b-a)]=a$，$b(a+b)-a=2ab+(b-a)$。（郭）

[2] 此求 $(a+b)^2+c^2+(b-a)^2+(ab+a+b)$。（郭）

[3] 开数即所求数。（郭）开方式的现代形式为：$z^2-96z+188=0$。（陈）

【今译】

今有长方形的阔乘面积，除以长，所得，减去阔与3倍的长阔差之和，余数与阔相等。只云：长乘长阔和，减去阔，等于2倍的长方形面积与长阔差之和。问：长阔和幂、弦幂、长阔差幂，连带长方形面积、长、阔六者共为多少？

答：94步。

术：设天元一为勾，地元一为股，人元一为所求数，天元、地元与人元三者相配合以求其解。得到188为常数项，−96为一次项系数，1为最高次项系数，开平方，即得。符合所问。

4.

【原文】

今有长乘积，减二平，如平而一，所得，加二较，如长而一，减平，与一较等。只云长乘较，减于直积，余如长而一，与二较同。[1]问：积幂、和

zhi ji, length, and width, and the squares of the *he*, *xian* and *jiao*.[2]

Ans. 94 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the required sum. Using these three talents we obtain 188 for the positive *shi*, 96 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[3] whose root is the required sum.

【Notes】

[1] That is, $\frac{a(ab)}{b} - [a + 3(b - a)] = a, b(a + b) - a = 2ab + (b - a)$. (G)

[2] That is, $(a + b)^2 + c^2 + (b - a)^2 + (ab + a + b)$. (G)

[3] The expression in modern form is the equation: $z^2 - 96z + 188 = 0$. (C)

4. From the product of the length and the *zhi ji* subtract twice the width and divide the remainder by the width; add to the quotient twice the *jiao* and divide this sum by the length then subtract the width; the result is equal to the *jiao*. From the *zhi ji* subtract the product of the length and the *jiao* and divide

幂、长幂、平幂、较幂，带一和、二平、三长、四较九事连环得几何？[2]

答曰：二百四十八步。

术曰：立天元一为勾，地元一为股，人元一为开数，三才相配求之。得七万九千六百八为正实，五百六十九为益方，一为正隅，平方开之。[3] 合问。

【注释】

[1]此即：$\dfrac{\dfrac{b(ab)-2a}{a}+2(b-a)}{b}-a=b-a$，$\dfrac{ab-b(b-a)}{b}=2(b-a)$。（郭）

[2]此求 $(ab)^2+(a+b)^2+b^2+a^2+(b-a)^2+[(a+b)+2a+3b+4(b-a)]$。（郭）

[3]开方式的现代形式为：$z^2-569z+79608=0$。（陈）

【今译】

今有长方形的长乘面积，减二阔，除以阔，所得，加2倍的长阔差，除以长，减阔，余数与长阔差相等。只云：长乘长阔差，减于长方形面积，余数除以长，等于2倍的长阔差。问：长方形面积之幂、长阔和幂、长幂、阔幂、长阔差幂，连带1倍的长阔和、2倍的阔、3倍的长、4倍的长阔差九者共为多少？

答：248步。

术：设天元一为勾，地元一为股，人元一为所求数，天元、地元与人元三者相配合以求其解。得到79608为常数项，−569为一次项系数，1为最高次项系数，开平方，即得。符合所问。

the remainder by the length; the quotient is equal to twice the *jiao*. [1] Find the sum of the nine matters: the *he*, twice the width, three times the length, four times the *jiao*, and the squares of the *zhi ji*, *he*, length, width, and *jiao*[2].

Ans. 248 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the sum of the nine matters. Using these three talents we obtain 79608 for the positive *shi*, 569 for the negative *fang*, and 1 for the positive *yu*, a quadratic expression[3] whose root is the required sum.

【Notes】

[1] That is, $\dfrac{\dfrac{b(ab)-2a}{a}+2(b-a)}{b}-a=b-a$, $\dfrac{ab-b(b-a)}{b}=2(b-a)$. (G)

[2] That is, $(ab)^2+(a+b)^2+b^2+a^2+(b-a)^2+[(a+b)+2a+3b+4(b-a)]$. (G)

[3] The expression in modern form is the equation: $z^2-569z+79608=0$. (C)

5.

【原文】

今有直积，减勾股和，以勾乘之，加勾股较，减直积，以股除之，加勾股和，以股乘之，加股，以勾除之，减二股、一勾股较，与勾等。只云勾乘股幂，以股除之，加直积，减勾幂，以勾除之，与弦同。[1] 问：五和、五较，带一积、一弦共一十二事连环得几何？[2]

答曰：六十九步。

术曰：立天元一为勾，地元一为股，人元一为开数，三才相配求之。得一万五千一百八十为正实，二千八十三为益方，二十七为正隅，平方开之。[3] 合问。

【注释】

[1] 此即：$\dfrac{\left\{\dfrac{[ab-(a+b)]a+(b-a)-ab}{b}+(a+b)\right\}b+b}{a}-[2b+(b-a)]=a$，$\dfrac{\dfrac{ab^2}{b}+ab-a^2}{a}=c$。（郭）

[2] 记五和之和 $(a+b)+(a+c)+(b+c)+[(a+b)+c]+[c+(b-a)]$ 为 A，五较之和 $(b-a)+(c-a)+(c-b)+[c-(b-a)]+[(a+b)-c]$ 为 B，此即求 $A+B+ab+c$。（郭）

[3] 开方式的现代形式为：$27z^2-2083z+15180=0$。（陈）

【今译】

今有长方形的面积，减勾股和，乘以勾，加勾股较，减长方形面积，除以股，加勾股和，乘以股，加股，除以勾，减去2倍的股与1倍的勾股较，与勾相等。只云：勾乘股幂，除以股，加长方形面积，减勾幂，除

5. From the *zhi ji* subtract the sum of the *gou* and the *gu* and multiply the remainder by the *gou*, add the *gu* less the *gou* then subtract the *zhi ji* and divide the result by the *gu*; add to the quotient the *gu* plus the *gou* and multiply the sum by the *gu*; to this product add the *gu* and divide the result by the *gou*; subtract from the quotient the sum of twice the *gu* and the difference between the *gu* and the *gou*; the result is equal to the *gou*. Divide the product of the *gou* and the square of the *gu* by the *gu*, add to the quotient the *zhi ji* and subtract the square of the *gou*, then divide the remainder by the *gou*; the result is equal to the *xian*. [1] Find the five *he*, the five *jiao*, the *zhi ji*, and the *xian*, the twelve matters taken together.[2]

Ans. 69 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the sum of the twelve matters. Using these three talents we obtain 15180 for the positive *shi*, 2083 for the negative *fang*, and 27 for the positive *yu*, a quadratic expression[3] whose root is the required sum.

[Notes]

[1] That is, $\dfrac{\left\{\dfrac{[ab-(a+b)]a+(b-a)-ab}{b}+(a+b)\right\}b+b}{a}-[2b+(b-a)]=a$, $\dfrac{\dfrac{ab^2}{b}+ab-a^2}{a}=c$. (G)

[2] Let the sum of five *he*, $(a+b)+(a+c)+(b+c)+[(a+b)+c]+[c+(b-a)]$, be A. Let the sum of five *jiao*, $(b-a)+(c-a)+(c-b)+[c-(b-a)]+[(a+b)-c]$, be B. That is, $A+B+ab+c$. (G)

[3] The expression in modern form is the equation: $27z^2-2083z+15180=0$. (C)

以勾，与弦相等。问：句股和、句弦和、股弦和、弦和和、弦较和这五和，及勾股较、句弦较、股弦较、弦较较、弦和较这五较，连带1倍的长方形面积、1倍的弦十二项之和共为多少？

答：69步。

术：设天元一为勾，地元一为股，人元一为所求数，天元、地元与人元三者相配合以求其解。得到15180为常数项，−2083为一次项系数，27为最高次项系数，开平方，即得。符合所问。

6.

【原文】

今有直积，减勾股和，以勾乘之，加勾股较，减直积，以股除之，加勾股和，以股乘之，加股，以勾除之，减二股、一勾股较，与勾等。只云勾乘股幂，以股除之，加直积，减勾幂，以勾除之，与弦同。问：弦较较乘直积得几何？

答曰：四十八步。

术曰：立天元一为勾，地元一为股，人元一为开数，三才相配求之。得六千为正实，四万六千七百八十一为益方，九百七十二为从隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$972z^2-46781z+6000=0$。（陈）其根为$[c-(b-a)]ab$。（郭）

【今译】

今有长方形的面积，减勾股和，乘以勾，加勾股较，减长方形面积，除以股，加勾股和，乘以股，加股，除以勾，减去2倍的股与1倍的勾股较，与勾相等。只云：勾乘股幂，除以股，加长方形面积，减勾幂，除以勾，与弦相等。问：弦较较乘长方形面积为多少？

答：48步。

术：设天元一为勾，地元一为股，人元一为所求数，天元、地元与人元三者相配合以求其解。得到6000为常数项，−46781为一次项系数，972为最高次项系数，开平方，即得。符合所问。

6. From the *zhi ji* subtract the sum of the *gou* and the *gu* and multiply the remainder by the *gou*, add the *gu* less the *gou* then subtract the *zhi ji* and divide the result by the *gu*; add to the quotient the *gou* plus the *gu* and multiply the sum by the *gu*; to this product add the *gu* and divide the result by the *gou*; subtract from the quotient the sum of twice the *gu* and the difference between the *gu* and the *gou*; the result is equal to the *gou*. Divide the product of the *gou* and the square of the *gu* by the *gu*, add to the quotient the *zhi ji* and subtract the square of the *gou*, then divide the remainder by the *gou*; the result is equal to the *xian*. Find the product of the *zhi ji* by the *xian* less the *jiao*.

Ans. 48 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the required number. Using these three talents we obtain 6000 for the positive *shi*, 46781 for the negative *fang*, and 972 for the positive *yu*, a quadratic expression[1] whose root is the required number.

[Notes]

[1] The expression in modern form is the equation: $972z^2 - 46781z + 6000 = 0$. (C) The square root is $[c - (b - a)]ab$. (G)

7.

【原文】

今有直积，减勾股和，以勾乘之，加勾股较，减直积，以股除之，加勾股和，以股乘之，加股，以勾除之，减二股、一勾股较，与勾等。只云勾乘股幂，以股除之，加直积，减勾幂，以勾除之，与弦同。问：弦和和乘黄方得几何？

答曰：二十四步。

术曰：立天元一为勾，地元一为股，人元一为开数，三才相配求之。得六百为益实，一千三百二十一为从方，五十四为益隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$-54z^2+1321z-600=0$。（陈）其根为$[(a+b)+c][(a+b)-c]$。罗士琳认为此问舛误。（郭）

【今译】

今有长方形的面积，减勾股和，乘以勾，加勾股较，减长方形面积，除以股，加勾股和，乘以股，加股，除以勾，减去2倍的股与1倍的勾股较，与勾相等。只云：勾乘股幂，除以股，加长方形面积，减勾幂，除以勾，与弦相等。问：弦和和乘黄方为多少？

答：24步。

术：设天元一为勾，地元一为股，人元一为所求数，天元、地元与人元三者相配合以求其解。得到-600为常数项，1321为一次项系数，-54为最高次项系数，开平方，即得。符合所问。

7. From the *zhi ji* subtract the sum of the *gou* and the *gu* and multiply the remainder by the *gou*, add the *gu* less the *gou* then subtract the *zhi ji* and divide the result by the *gu*; add to the quotient the *gu* plus the *gou* and multiply the sum by the *gu*; to this product add the *gu* and divide the result by the *gou*; subtract from the quotient the sum of twice the *gu* and the difference between the *gu* and the *gou*; the result is equal to the *gou*. Divide the product of the *gou* and the square of the *gu* by the *gu*, add to the quotient the *zhi ji* and subtract the square of the *gou*, then divide the remainder by the *gou*; the result is equal to the *xian*. Find the product of the *huang fang* by the *xian* plus the *he*.

Ans. 24 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the required number. Using these three talents we obtain 600 for the negative *shi*, 1321 for the positive *fang*, and 54 for the negative *yu*, a quadratic expression[1] whose root is the required number.

[Notes]

[1] The expression in modern form is the equation: $-54z^2 + 1321z - 600 = 0$. (C) The square root is $[(a + b) + c][(a + b) - c]$. Luo Shilin considered that the problem was mistaken. (G)

8.

【原文】

今有直积，减勾股和，以勾乘之，加勾股较，减直积，以股除之，加勾股和，以股乘之，加股，以勾除之，减二股、一勾股较，与勾等。只云勾乘股幂，以股除之，加直积，减勾幂，以勾除之，与弦同。问：五和乘五较得几何？

答曰：四百二十步。

术曰：立天元一为勾，地元一为股，人元一为开数，三才相配求之。得三十六万七千五百为正实，四万六千二百三十五为益方，一百八为正隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$108z^2 - 46235z + 367500 = 0$。（陈）其根为 $A \times B$。（郭）

【今译】

今有长方形的面积，减勾股和，乘以勾，加勾股较，减长方形面积，除以股，加勾股和，乘以股，加股，除以勾，减去2倍的股与1倍的勾股较，与勾相等。只云：勾乘股幂，除以股，加长方形面积，减勾幂，除以勾，与弦相等。问：五和乘五较为多少？

答：420步。

术：设天元一为勾，地元一为股，人元一为所求数，天元、地元与人元三者相配合以求其解。得到367500为常数项，−46235为一次项系数，108为最高次项系数，开平方，即得。符合所问。

8. From the *zhi ji* subtract the sum of the *gou* and the *gu* and multiply the remainder by the *gou*, add the *gu* less the *gou* then subtract the *zhi ji* and divide the result by the *gu*; add to the quotient the *gu* plus the *gou* and multiply the sum by the *gu*; to this product add the *gu* and divide the result by the *gou*; subtract from the quotient the sum of twice the *gu* and the difference between the *gu* and the *gou*; the result is equal to the *gou*. Divide the product of the *gou* and the square of the *gu* by the *gu*, add to this quotient the *zhi ji* and subtract the square of the *gou*, then divide the remainder by the *gou*; the result is equal to the *xian*. Find the product of the five *he* by the five *jiao*.

Ans. 420 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the required number. Using these three talents we obtain 367500 for the positive *shi*, 46235 for the negative *fang*, and 108 for the positive *yu*, a quadratic expression[1] whose root is the required number.

[Notes]

[1] The expression in modern form is the equation: $108z^2 - 46235z + 367500 = 0$. (C) The square root is $A \times B$. (G)

9.

【原文】

今有直积，减勾股和，以勾乘之，加勾股较，减直积，以股除之，加勾股和，以股乘之，加股，以勾除之，减二股、一勾股较，与勾等。只云勾乘股幂，以股除之，加直积，减勾幂，以勾除之，与弦同。问：五较连环除弦，乘三相和得几何？

答曰：六步。

术曰：立天元一为勾，地元一为股，人元一为开数，三才相配求之。得三十为正实，四十一为益方，六为正隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$6z^2-41z+30=0$。(陈) 其根为 $\frac{C}{B}\times(a+b+c)$。(郭)

【今译】

今有长方形的面积，减勾股和，乘以勾，加勾股较，减长方形面积，除以股，加勾股和，乘以股，加股，除以勾，减去2倍的股与1倍的勾股较，与勾相等。只云：勾乘股幂，除以股，加长方形面积，减勾幂，除以勾，与弦相等。问：弦除以五较之和，乘以勾、股、弦三者之和为多少？

答：6步。

术：设天元一为勾，地元一为股，人元一为所求数，天元、地元与人元三者相配合以求其解。得到30为常数项，−41为一次项系数，6为最高次项系数，开平方，即得。符合所问。

9. From the *zhi ji* subtract the sum of the *gou* and the *gu* and multiply the remainder by the *gou*, add the *gu* less the *gou* then subtract the *zhi ji* and divide the result by the *gu*; add to the quotient the *gu* plus the *gou* and multiply the sum by the *gu*; to this product add the *gu* and divide the result by the *gou*; subtract from the quotient the sum of twice the *gu* and the difference between the *gu* and the *gou*; the result is equal to the *gou*. Divide the product of the *gou* and the square of the *gu* by the *gu*, add to the quotient the *zhi ji* and subtract the square of the *gou*, then divide the remainder by the *gou*; the result is equal to the *xian*. Find the product of the *xian* by the sum of the sides divided by the *five jiao*.

Ans.6 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the required number. Using these three talents we obtain 30 for the positive *shi*, 41 for the negative *fang*, and 6 for the positive *yu*, a quadratic expression[1] whose root is the required number.

【Notes】

[1] The expression in modern form is the equation: $6z^2 - 41z + 30 = 0$. (C) The square root is $\frac{C}{B} \times (a + b + c)$. (G)

10.

【原文】

今有直积，减勾股和，以勾乘之，加勾股较，减直积，以股除之，加勾股和，以股乘之，加股，以勾除之，减二股、一勾股较，与勾等。只云勾乘股幂，以股除之，加直积，减勾幂，以勾除之，与弦同。问：五和、五较、勾、股及弦十三事连环除股，乘直积幂得几何？

答曰：九步。

术曰：立天元一为勾，地元一为股，人元一为开数，三才相配求之。得五千六百二十五为正实，一百六十八万二百四十一为益方，一十八万六千六百二十四为正隅，平方开之。[1] 合问。

【注释】

[1] 开方式的现代形式为：$186624z^2 - 1680241z + 5625 = 0$。（陈）其根为 $\frac{b}{A+B+a+b+c} \times ab$。（郭）

【今译】

今有长方形的面积，减勾股和，乘以勾，加勾股较，减长方形面积，除以股，加勾股和，乘以股，加股，除以勾，减去2倍的股与1倍的勾股较，与勾相等。只云：勾乘股幂，除以股，加长方形面积，减勾幂，除以勾，与弦相等。问：股除以五和、五较、勾、股、弦十三项之和，乘以长方形面积，得到多少？

答：9步。

术：设天元一为勾，地元一为股，人元一为所求数，天元、地元与人元三者相配合以求其解。得到5625为常数项，−1680241为一次项系数，186624为最高次项系数，开平方，即得。符合所问。

10. From the *zhi ji* subtract the sum of the *gou* and the *gu* and multiply the remainder by the *gou*, add the *gu* less the *gou* then subtract the *zhi ji* and divide the result by the *gu*; add to the quotient the *gu* plus the *gou* and multiply the sum by the *gu*; to this product add the *gu* and divide the result by the *gou*; subtract from the quotient the sum of twice the gu and the difference between the *gu* and the *gou*; the result is equal to the *gou*. Divide the product of the *gou* and the square of the *gu* by the *gu*, add to the quotient the *zhi ji* and subtract the square of the *gou*, then divide the remainder by the *gou*; the result is equal to the *xian*. Find the product of the *gu* by the square of the *zhi ji* divided by the sum of the thirteen matters: the five *he*, the five *jiao*, the *gou*, *gu*, and *xian*.

Ans.9 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the required number. Using these three talents we obtain 5625 for the positive *shi*, 1680241 for the negative *fang*, and 186624 for the positive *yu*, a quadratic expression[1] whose root is the required number.

[Notes]

[1] The expression in modern form is the equation: $186624z^2 - 1680241z + 5625 = 0$. (C) The square root is $\frac{b}{A + B + a + b + c} \times ab$. (G)

11.

【原文】

今有勾弦较乘股弦和，加勾股和，开方，得数多股一步。只云股弦较乘弦较和，减弦，开方，得数少黄方幂三步。[1] 问：弦和较几何？

答曰：二步。

术曰：立天元一为勾，地元一为股，人元一为弦和较，三才相配求之。得三千五百六十为正实，七百六十为从方，七千六百九十八为益上廉，一千一百三十四为益二廉，六千五百二十二为从三廉，七百二十为从四廉，二千八百二十九为益五廉，二百三十三为益六廉，六百七十为从七廉，三十六为从八廉，八十二为益九廉，二为益十廉，四为正隅，十一乘方开之。[2] 合问。

【注释】

[1] 此即：$\sqrt{(c-a)(b+c)+(a+b)}-b=1$；并且

$[(a+b)-c]^2-\sqrt{(c-b)[c+(b-a)]-c}=3$。（郭）

[2] 开方式的现代形式为：

$4x^{12}-2x^{11}-82x^{10}+36x^9+670x^8-233x^7-2829x^6+720x^5+6522x^4-1134x^3-7698x^2+760x+3560=0$。（陈）

【今译】

今有勾弦较乘股弦和，加勾股和，开平方，其得数比股多 1 步。只云：股弦较乘弦较和，减去弦，开平方，其得数比黄方之幂少 3 步。问：弦和较为多少？

答：2 步。

术：设天元一为勾，地元一为股，人元一为弦和较，天元、地元与人元三者相配合以求其解。得到 3560 为常数项，760 为一次项系数，−7698 为二次项系数，−1134 为三次项系数，6522 为四次项系数，720 为五次项系数，−2829 为六次项系数，−233 为七次项系数，670 为八次项系数，36 为九次项系数，−82 为十次项系数，−2 为十一次项系数，4 为最高次项系数，开十二次方。符合所问。

11. To the product of the *xian* less the *gou* by the *xian* plus the *gu* add the *gou* plus the *gu*; the square root of the result exceeds the *gu* by 1 *bu*. From the product of the *xian* less the *gu* by the *xian* plus the *jiao* subtract the *xian*; the square root of the result is less than the square of the *huang fang* by 3 *bu*. [1] Find the *he* less the *xian*.

Ans.2 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, and the element *ren* the *he* less the *xian*. Using these three talents we obtain 3560 for the positive *shi*, 760 for the positive *fang*, 7698 for the negative upper *lian*, 1134 for the negative second *lian*, 6522 for the positive third *lian*, 720 for the positive fourth *lian*, 2829 for the negative fifth *lian*, 233 for the negative sixth *lian*, 670 for the positive seventh *lian*, 36 for the positive eighth *lian*, 82 for the negative ninth *lian*, 2 for the negative tenth *lian*, and 4 for the positive *yu*, an expression[2] of the twelfth degree whose root is the required number.

【Notes】

[1] That is, $\sqrt{(c-a)(b+c)+(a+b)}-b=1$; and

$[(a+b)-c]^2-\sqrt{(c-b)[c+(b-a)]-c}=3$. (G)

[2] The expression in modern form is the equation: $4x^{12}-2x^{11}-82x^{10}+36x^9+670x^8-233x^7-2829x^6+720x^5+6522x^4-1134x^3-7698x^2+760x+3560=0$. (C)

四象朝元 六问

1.

【原文】

今有五平、三长，立方开之，少股一步。只云三和、四较，平方开之，多勾二步。[1] 问：弦较较带黄方二事连环得几何？

答曰：六步。

术曰：立天元一为勾，地元一为股，人元一为弦，物元一为开数，四象和会求之。得三万九千三百六十为正实，七万四千八十为从方，五千五百二十为从上廉，二百八十为益二廉，二百六十四为益三廉，三十为益四廉，一为益隅，五乘方开之。[2] 合问。

【注释】

[1] 此即：$b-\sqrt[3]{5a+3b}=1$，$\sqrt{3(a+b)+4(b-a)}-a=2$。（郭）

[2] 开方式的现代形式为：$-u^6-30u^5-264u^4-280u^3+5520u^2+74080u+39360=0$。（陈）

【今译】

今有长方形的5倍的阔与3倍的长之和，开立方，其得数比股少1步。只云：3倍的长阔和与4倍的长阔差之和，开平方，其得数比勾多2步。问：弦较较与黄方二者之和为多少？

答：6步。

术：设天元一为勾，地元一为股，人元一为弦，物元一为所求数，天元、地元、人元、物元四者相和会以求其解。得到39360为常数项，74080为一次项系数，5520为二次项系数，−280为三次项系数，−264为四次项系数，−30为五次项系数，−1为最高次项系数，开六次方。符合所问。

Si Xiang Chao Yuan (Expressions in Four Unknown Quantities)

6 Problems

1. The cube root of the sum of five times the width and three times the length is less than the *gu* by 1 *bu*; the square root of the sum of three times the *he* and four times the *jiao* exceeds the *gou* by 2 *bu*. [1] Find the sum of the two matters: the *huang fang* and the difference between the *xian* and the *jiao*.

Ans.6 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, the element *ren* the *xian*, and the element *wu* the required number. These four phenomena meeting we obtain 39360 for the positive *shi*, 74080 for the positive *fang*, 5520 for the positive upper *lian*, 280 for the negative second *lian*, 264 for the negative third *lian*, 30 for the negative fourth *lian*, and 1 for the negative *yu*, an expression[2] of the sixth degree whose root is the required number.

[Notes]

[1] That is, $b - \sqrt[3]{5a + 3b} = 1, \sqrt{3(a + b) + 4(b - a)} - a = 2$. (G)

[2] The expression in modern form is the equation: $-u^6 - 30u^5 - 264u^4 - 280u^3 + 5520u^2 + 74080u + 39360 = 0$. (C)

2.

【原文】

今有弦较和如股幂八分之三。只云弦较较幂如勾弦和幂四分之一。[1]问：二弦、四勾、二股三事连环得几何？

答曰：三十步。

术曰：立天元一为勾，地元一为股，人元一为弦，物元一为开数，四象和会求之。得六万为益实，一千一百为从方，三百为从廉，九为益隅，立方开之。[2] 合问。

【注释】

[1] 此即：$c+(b-a)=\frac{3}{8}b^2$，$[c-(b-a)]^2=\frac{1}{4}(a+c)^2$。（郭）

[2] 开方式的现代形式为：$-9u^3+300u^2+1100u-60000=0$。（陈）

【今译】

今有弦较和等于股幂的$\frac{3}{8}$。只云：弦较较幂等于勾弦和幂的$\frac{1}{4}$。问：2倍的弦、4倍的勾、2倍的股三者之和为多少？

答：30步。

术：设天元一为勾，地元一为股，人元一为弦，物元一为所求数，天元、地元、人元、物元四者相和会以求其解。得到-60000为常数项，1100为一次项系数，300为二次项系数，-9为最高次项系数，开立方。符合所问。

3.

【原文】

今有弦和较幂加勾弦较幂，比股幂少一段勾弦和。只云弦和和幂减勾股

2. The sum of the *xian* and the *jiao* is equal to three-eighths of the square of the *gu*; the square of the difference between the *xian* and the *jiao* is equal to one-fourth of the square of the sum of the *xian* and the *gou*. [1] Find the sum of the three matters: twice the *xian*, four times the *gou*, and twice the *gu*.

Ans.30 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, the element *ren* the *xian*, and the element *wu* the required number. These four phenomena meeting we obtain 60000 for the negative *shi*, 1100 for the positive *fang*, 300 for the positive *lian*, and 9 for the negative *yu*, a cubic expression [2] whose root is the required number.

[Notes]

[1] That is, $c + (b - a) = \frac{3}{8}b^2$, $[c - (b - a)]^2 = \frac{1}{4}(a + c)^2$. (G)

[2] The expression in modern form is the equation: $-9u^3 + 300u^2 + 1100u - 60000 = 0$. (C)

3. The square of the difference between the *he* and the *xian* plus the square of the difference between the *xian* and the *gou* is less than the square of the *gu* by the sum of the *gou* and the *xian*. The square of the sum of the *xian* and the *he*,

和，乘弦和和，加勾弦较，比二弦幂多一段三相和。[1] 问：勾、股、弦三事连环得几何？

答曰：一十二步。

术曰：立天元一为勾，地元一为股，人元一为弦，物元一为开数，四象和会求之。得四十八为益实，四百二十四为从方，八百五十一为益上廉，六十八为从二廉，九百六十为从三廉，六百五十六为益四廉，四十八为从隅，五乘方开之。[2] 合问。

【注释】

[1] 此即：$b^2-\{[(a+b)-c]^2+(c-a)^2\}=c+a$，$[c+(a+b)]^2-(a+b)[c+(a+b)]+(c-a)-2c^2=a+b+c$。（郭）

[2]“九百六十”原文讹作“九十六”，依戴煦校正。此即六次方程：$48u^6-656u^5+960u^4+68u^3-851u^2+424u-48=0$。（陈）

【今译】

今有弦和较幂加勾弦较幂，比股幂少一勾弦和。只云：弦和和幂减去勾股和乘弦和和，加勾弦较，比2倍的弦幂多勾、股、弦三者之和。问：勾、股、弦三者之和为多少？

答：12步。

术：设天元一为勾，地元一为股，人元一为弦，物元一为所求数，天元、地元、人元、物元四者相和会以求其解。得到-48为常数项，424为一次项系数，-851为二次项系数，68为三次项系数，960为四次项系数，-656为五次项系数，48为最高次项系数，开六次方。符合所问。

less the sum of the *gou* and the *gu* multiplied by the sum of the *xian* and the *he*, plus the difference between the *xian* and the *gou* exceeds twice the square of the *xian* by the sum of the three [sides].[1] Find the sum of the three matters: the *gou*, *gu*, and *xian*.

Ans. 12 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, the element *ren* the *xian*, and the element *wu* the required number. These four phenomena meeting we obtain 48 for the negative *shi*, 424 for the positive *fang*, 851 for the negative upper *lian*, 68 for the positive second *lian*, 960 for the positive third *lian*, 656 for the negative fourth *lian*, and 48 for the positive *yu*, an expression[2] of the sixth degree whose root is the required number.

[Notes]

[1] That is, $b^2 - \{[(a+b)-c]^2 + (c-a)^2\} = c + a$, $[c + (a+b)]^2 - (a+b)[c + (a+b)] + (c-a) - 2c^2 = a + b + c$. (G)

[2] 96 in the original text is an error. According to Dai Xu's research, we substitute 960 for 96. The expression in modern form is the equation: $48u^6 - 656u^5 + 960u^4 + 68u^3 - 851u^2 + 424u - 48 = 0$. (C)

4.

【原文】

今有五和并三事，与四直积加二勾等。只云三事减五较，与四相和常例：五和、五较。四和者：句、股、弦、黄方。减股幂同。[1] 问：句、股、弦、和、较五事连环得几何？

答曰：二十步。

术曰：立天元一为勾，地元一为股，人元一为弦，物元一为开数，四象和会求之。得七千六百八十为正实，七万五千五百八十四为益方，一十一万八千一百六十为从上廉，五千七百八十为从二廉，二千三百七十五为益三廉，七十为益四廉，八为正隅，五乘方开之。[2] 合问。

【注释】

[1] 记五和、五较分别为A，B，此即：$A+(a+b+c)=4ab+2a$，$(a+b+c)-B=\{a+b+c+[(a+b)-c]\}-b^2$。（郭）

[2] 开方式的现代形式为：$8u^6-70u^5-2375u^4+5780u^3+118160u^2-75584u+7680=0$。（陈）

【今译】

今有五和加勾、股、弦之和，与4倍的长方形面积加2倍的勾相等。只知道：勾、股、弦之和减去五较和，与四相之和五和、五较按常例。四相者为勾、股、弦、黄方。减去股幂相等。问：勾、股、弦、勾股和、勾股较五者之和为多少？

答：20步。

术：设天元一为勾，地元一为股，人元一为弦，物元一为所求数，天元、地元、人元、物元四者相和会以求其解。得到7680为常数项，−75584为一次项系数，118160为二次项系数，5780为三次项系数，−2375为四次项系数，−70为五次项系数，8为最高次项系数，开六次方。符合所问。

4. The five *he* plus the sum of the three [sides] is equal to four times the *zhi ji* plus twice the *gou*. The difference between the sum of the three [sides] and the five *jiao* is equal to the sum of the four [the *gou*, *gu*, *xian*, and the *huang fang*] minus the square of the *gu*.[1] Find the sum of the five matters: the *gou*, *gu*, *xian*, *he* and *jiao*.

Ans.20 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, the element *ren* the *xian*, and the element *wu* the required number. These four phenomena meeting we obtain 7680 for the positive *shi*, 75584 for the negative *fang*, 118160 for the positive upper *lian*, 5780 for the positive second *lian*, 2375 for the negative third *lian*, 70 for the negative fourth *lian*, and 8 for the positive *yu*, an expression[2] of the sixth degree whose root is the required number.

[Notes]

[1] Let the five *he* be A, and the five *jiao* B. From the statement we have $A + (a + b + c) = 4ab + 2a$, $(a + b + c) - B = \{a + b + c + [(a + b) - c]\} - b$. (G)

[2] The expression in modern form is the equation: $8u^6 - 70u^5 - 2375u^4 + 5780u^3 + 118160u^2 - 75584u + 7680 = 0$. (C)

5.

【原文】

今有勾、股、弦各自乘，减五和，与倍之黄方幂适等。只云：五和加二勾、二股幂，减八弦，与半之五较幂多弦和较相同。[1] 问：立方开十三事得几何？

答曰：四步。

术曰：立天元一为勾，地元一为股，人元一为弦，物元一为开数，四象和会求之。得三千七十二为正实，一百一十二为益方，四十一为益隅，立方开之，[2] 得四步。合问。

【注释】

[1] 此即：$(a^2+b^2+c^2)-A=2[(a+b)-c]^2$，

$(A+2a^2+2b^2)-8c=\frac{1}{2}B^2+[(a+b)-c]$。（郭）

[2] 开方式的现代形式为：$-41u^3-112u+3072=0$。（陈）

【今译】

今有勾、股、弦各自乘之和，减去五和，与2倍的黄方幂相等。只云：五和加2倍的勾幂与股幂之和，减8倍的弦，等于五较幂的$\frac{1}{2}$与弦和较之和。问：勾、股、弦与五和、五较之和，立方开之为多少？

答：4步。

术：设天元一为勾，地元一为股，人元一为弦，物元一为所求数，天元、地元、人元、物元四者相和会以求其解。得到3072为常数项，−112为一次项系数，−41为最高次项系数，开立方，得4步。符合所问。

5. The sum of the squares of the *gou*, the *gu*, and the *xian* minus the five *he* is equal to twice the square of the *huang fang*. The five *he* plus twice the square of the *gou* plus twice the square of the *gu* minus eight times the *xian* is equal to one-half of the square of the five *jiao* plus the difference between the *he* and the *xian*.[1] Find the cube root of the sum of the thirteen matters: the *gou*, *gu*, *xian*, five *he*, and five *jiao*.

Ans.4 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, the element *ren* the *xian*, and the element *wu* the required number. These four phenomena meeting we obtain 3072 for the positive *shi*, 112 for the negative *fang*, and 41 for the negative *yu*, a cubic expression[2] whose root, 4 *bu*, is the required number.

【Notes】

[1] That is, $(a^2 + b^2 + c^2) - A = 2[(a + b) - c]^2$, $(A + 2a^2 + 2b^2) - 8c = \frac{1}{2}B^2 + [(a + b) - c]$. (G)

[2] The expression in modern form is the equation: $-41u^3 - 112u + 3072 = 0$. (C)

6.

【原文】

今有一数不知多少，但言：五较各自乘，并之为正实，以三为益方，一为从上廉，一为从下廉，二为益隅，三乘方开之，与其数相等。[1] 只云：勾股和幂减二直积，加三相和，与其数幂自乘、并弦幂、减股相同。[2] 又云：半之三相和，加黄方，与其数再自乘亦等。[3] 问：元数几何？

答曰：二步。

术曰：立天元一为勾，地元一为股，人元一为弦，物元一为开数，四象和会求之。得一千一百五十二为正实，七百六十八为益方，六百四十为益上廉，一千七百九十二为从二廉，三百八十四为益三廉，九千八为益四廉，一万九千一百一十二为从五廉，八千七百九十九为益六廉，八千七百九十五为益七廉，一万二千六百三十七为从八廉，二千三十为从九廉，一万九千一百六十八为益十廉，二万二千二百九十二为从十一廉，一万一千一百一十二为益十二廉，二千六为正隅，十三乘方开之，[4] 得二步，即元数也。合问。

【注释】

[1] 记五较各自乘之和为 Q：

$Q=(a+b)^2+(a+c)^2+(b+c)^2+[(a+b)+c]^2+[(b-a)+c]^2$，又记所求数为 u，则 u 满足开方式：$-2u^4+u^3+u^2-3u+Q=0$。（郭）

[2] 此即：$(a+b)^2-2ab+(a+b+c)=(u^2)^2+c^2-b$。（郭）

[3] 此即：$\frac{1}{2}(a+b+c)+[(a+b)-c]=u^3$。（郭）

[4] 开方式的现代形式为：$2006u^{14}-11112u^{13}+22292u^{12}-19168u^{11}+2030u^{10}+12637u^9-8795u^8-8799u^7+19112u^6-9008u^5-384u^4+1792u^3-640u^2-768u+1152=0$。（陈）

6. A number is to be determined from the following conditions. If we take the sum of the square of the five *jiao* for the positive *shi*, 3 for the negative *fang*, 1 for the positive upper *lian*, 1 for the positive lower *lian*, and 2 for the negative *yu*, we then form an expression of the fourth degree whose root is the required number. [1] The square of the sum of the *gou* and the *gu* minus twice the *zhi ji* plus the sum of the three [sides] is equal to the square of the square of the required number plus the square of the *xian* minus the *gu*; [2] and one-half of the sum of the three [sides] plus the *huang fang* is equal to the cube of the required number. [3] What is the number?

Ans. 2 *bu*.

Process. Let the element *tian* be the *gou*, the element *di* the *gu*, the element *ren* the *xian*, and the element *wu* the required number. These four phenomena meeting we obtain 1152 for the positive *shi*, 768 for the negative *fang*, 640 for the negative upper *lian*, 1792 for the positive second *lian*, 384 for the negative third *lian*, 9008 for the negative fourth *lian*, 19112 for the positive fifth *lian*, 8799 for the negative sixth *lian*, 8795 for the negative seventh *lian*, 12637 for the positive eighth *lian*, 2030 for the positive ninth *lian*, 19168 for the negative tenth *lian*, 22292 for the positive eleventh *lian*, 11112 for the negative twelfth *lian*, and 2006 for the positive *yu*, an expression[4] of the fourteenth degree whose root, 2 *bu*, is the required number.

[Notes]

[1] Let the sum of the square of the five *jiao* be Q. Then, $Q = (a+b)^2 + (a+c)^2 + (b+c)^2 + [(a+b)+c]^2 + [(b-a)+c]^2$. And let the required number be u. Then, u satisfies the equation: $-2u^4 + u^3 + u^2 - 3u + Q = 0$. (G)

【今译】

今有一数不知多少，只说：五较各自乘，相加作为常数项，-3为一次项系数，1为二次项系数，1为三次项系数，-2为最高次项系数，开四次方，其开方数与该数相等。只知道：勾股和幂减去2倍的长方形面积，加勾、股、弦三者之和，与该数之幂的自乘，加弦幂，减股相等。又知道：勾、股、弦三者之和的$\frac{1}{2}$，加黄方，与该数的立方也相等。问：原来的数为多少？

答：2步。

术：设天元一为勾，地元一为股，人元一为弦，物元一为所求数，天元、地元、人元、物元四者相和会以求其解。得到1152为常数项，-768为一次项系数，-640为二次项系数，1792为三次项系数，-384为四次项系数，-9008为五次项系数，19112为六次项系数，-8799为七次项系数，-8795为八次项系数，12637为九次项系数，2030为十次项系数，-19168为十一次项系数，22292为十二次项系数，-11112为十三次项系数，2006为最高次项系数，开十四次方，得到2步，就是原来的数。符合所问。

[2] That is, $(a+b)^2 - 2ab + (a+b+c) = (u^2)^2 + c^2 - b$. (G)

[3] That is, $\frac{1}{2}(a+b+c) + [(a+b) - c] = u^3$. (G)

[4] The expression in modern form is the equation: $2006u^{14} - 11112u^{13} + 22292u^{12} - 19168u^{11} + 2030u^{10} + 12637u^9 - 8795u^8 - 8799u^7 + 19112u^6 - 9008u^5 - 384u^4 + 1792u^3 - 640u^2 - 768u + 1152 = 0$. (C)

附：朱世杰在《算学启蒙》中使用的度量单位换算表

【中译】

一、容 量

6粟 = 1圭

10圭 = 1撮

10撮 = 1抄

10抄 = 1勺

10勺 = 1合

10合 = 1升

10升 = 1斗

10斗 = 1斛

二、重 量

10黍 = 1絫

10絫 = 1铢

6铢 = 1分

4分 = 1两

16两 = 1斤

15斤 = 1秤

30斤 = 1钧

4钧 = 1硕 = 120斤

三、长　度

10忽＝1丝

10丝＝1毫

10毫＝1厘

10厘＝1分

10分＝1寸

10寸＝1尺

10尺＝1丈

3丈2尺＝1匹[1]，或者有时2丈4尺＝1匹

5丈5尺＝1端，或者有时4丈8尺＝1端

【注释】

[1]术语“匹”与“端”仅使用于布的度量中。布一般由单线编织，有时包括一根长线，有时也有其他情况。（陈）

Appendix: Tables of Measures Used by Zhu Shijie in His *Introduction to Mathematical Studies*

【Original Text】

I. Heaped Measure

6 *su*（粟, grain）= 1 *gui*

10 *gui*（圭）= 1 *cuo*（撮, pinch）

10 *cuo*（撮）= 1 *chao*（抄, handful）

10 *chao*（抄）= 1 *shao*（勺, spoonful）

10 *shao*（勺）= 1 *he*（合）

10 *he*（合）= 1 *sheng*（升）

10 *sheng*（升）= 1 *dou*（斗）

10 *dou*（斗）= 1 *hu*（斛）

II. Weight Measure

10 *shu*（黍）= 1 *lei*（絫）

10 *lei*（絫）= 1 *zhu*（铢）

6 *zhu*（铢）= 1 *fen*（分）

4 *fen*（分）= 1 *liang*（两）

16 *liang*（两）= 1 *jin*（斤）

15 *jin*（斤）= 1 *cheng*（秤）

30 *jin*（斤）= 1 *jun*（钧）

4 *jun*（钧）= 1 *shuo*（硕）= 120 *jin*（斤）

Ⅲ. Linear Measure

10 *hu* (忽) = 1 *si* (丝, silk thread)

10 *si* (丝) = 1 *hao* (毫)

10 *hao* (毫) = 1 *li* (厘)

10 *li* (厘) = 1 *fen* (分)

10 *fen* (分) = 1 *cun* (寸)

10 *cun* (寸) = 1 *chi* (尺)

10 *chi* (尺) = 1 *zhang* (丈)

3 *zhang* (丈) 2 *chi* (尺) = 1 *pi* (匹)[1] or sometimes 2 *zhang* (丈) 4 *chi* (尺) = 1 *pi* (匹)

5 *zhang* (丈) 5 *chi* (尺) = 1 *duan* (端) or sometimes 4 *zhang* (丈) 8 *chi* (尺) = 1 *duan* (端)

【Note】

[1] The terms *pi* and *duan* are used only in measuring cloth. Cloth is woven in single strips sometimes containing one length and sometimes another.(C)